LEARNING CONCEPTS THROUGH INQUIRY

Introduction

This unit considers how the processes of inquiry based learning may be integrated into the teaching of Mathematics and Science content. Often, these two aspects of learning are kept separate: we teach content as a collection of facts and skills to be imitated and mastered, and/or we teach process skills through investigations that do not develop incorporate important content knowledge. The integration of content and process raises many pedagogical challenges.

The processes under consideration here are: observing and visualising, classifying and creating definitions, making representations and translating between them, finding connections and relationships, estimating, measuring and quantifying, evaluating, experimenting and controlling variables. As some have pointed out, these are developments of natural human powers that we employ from birth (Millar, 1994). To some extent, we use them unconsciously all the time. When these powers are harnessed and developed by teachers to help students understand the concepts of mathematics and science, students become much more engaged and involved in their learning.

This unit has many activities within it - too many for one session. It is suggested that this unit is used as a menu, from which professional development providers can choose. It is however, important that participants are given an opportunity to try out some of these activities in their lessons and to report back on the outcomes.

Activities

Acknowledgement:
The modules have been compiled for PRIMAS from professional development materials developed by the Shell Centre team at the Centre for Research in Mathematics Education, University of Nottingham. This includes material adapted from *Improving Learning in Mathematics* © Crown Copyright (UK) 2005 by kind permission of the Learning and Skill Improvement Service www.LSIS.org.uk.

ACTIVITY A: OBSERVING AND VISUALISING

Time needed - 30 minutes

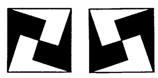
The processes of observing and visualising are natural human powers that we have from birth. Observation is primarily about what we can see and notice directly, whereas visualisation concerns what we can imagine and transform mentally, in our 'mind's eye'. The contention here is that these powers are often under-used in classrooms, at least partly because we don't use tasks that *require* the use of these powers for their successful completion.

The activities presented here are intended to be just examples of three ways of harnessing students' powers of observation and visualisation. These are only examples; alternatives are easily found at any level of difficulty. In the left hand column of the worksheet we offer generic descriptions of the activities, while in the right hand column we offer a specific example. These are discussed briefly, below.

- Work on some of the activities on **Handout 1**.
- Share you observations and mental images:
  - how did you *'see'* the object differently?
  - what did you notice *or single out for attention*?
  - what did you try to manipulate mentally?
- Try to develop an activity using one of these types for use in your own classroom. Try to devise examples that force students to observe properties carefully, and that will create discussion about definitions.
- Try out your activity and report back on it.

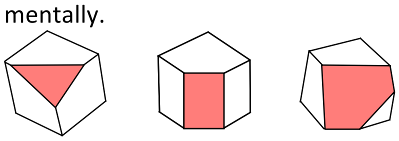
Alhambra

The Alhambra tiling is a complex repeating pattern made from many different shapes. Participants may be asked to sketch the individual tiles that went in to constructing it. Two small tiles will do, as shown below.
Could the pattern be made from one small tile?



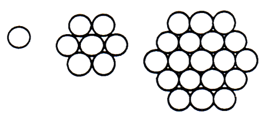
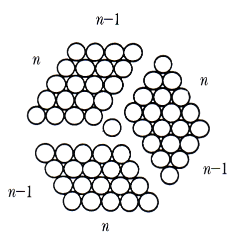
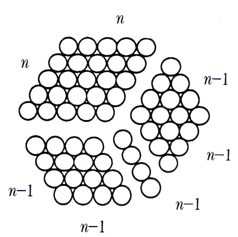
Cube of cheese

Ask participants to describe all the shapes that they 'see' as the cheese is cut. Initially a small triangle is formed, but this may be of any type, depending on the angle of the knife. As larger and larger cuts are made, participants may be surprised to 'see' all kinds of quadrilaterals, pentagons and hexagons. They may want to sketch diagrams and work on this further as they discuss. Encourage this, but only after trying to work mentally.



Suspension bridge cables

Different ways of seeing lead to different sequences and algebraic expressions:

			You may also be able to see the diagram as the difference of two cubes:
1, 7, 19, ...	$3n(n-1)+1$	$n^2+2(n-1)^2+(n-1)$	$n^3-(n-1)^3$

Handout 1: Observing and visualising activities

Describing what you see

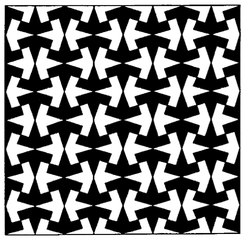
Show the class a poster or object and ask them to decribe what they can see as accurately as they can.

Sit two students back to back and give one of them a simple geometric design. As this person to describe the design so that the second person can reproduce it accurately.

Alhambra pattern

This tiling pattern may be found in the Alhambra palace in Granada, Spain.

- How would you describe this pattern to someone who cannot see it?
- Describe how individual tiles may have been constructed.



Visualising

Ask students to shut their eyes and imagine a situation in which something is changing. Ask them to describe what they 'see'.

Cube of cheese

Imagine you have a cube of cheese and a knife. Imagine you cut off one small corner of the cheese. What shape do you get?

Imagine cutting more and more parallel slices off the cheese. How will your triangle change? What shapes will be revealed? Keep going until there is no cheese left!
Now change the angle of your knife....

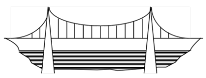
Looking for structure

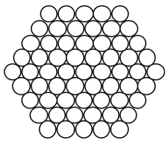
Give students a problem that encourages them to look for different structures within a context.
Ask them to use their structures to make generalisations.

In the example shown, they may be asked:

- In what different ways can you count the cables?
- Can you *see* the diagram in different ways?
    - Can you see it as composed of parallellograms or triangles ?
    - Can you see a 3 dimensional shape ?

Suspension bridge cables





When making a cable for a suspension bridge, many strands are assembled into a hexagonal formation and then 'compacted' together. This diagram illustrates a 'size 5' cable made up of 61 strands. How many strands are needed for a size 10 cable?
How many for a cable that is size n?

The *Alhambra pattern* task and the *Suspension bridge cables* task are both taken from Swan and Crust (1993) *Mathematics Programmes of Study, Inset for Key Stages 3 and 4*, National Curriculum Council, York.

Looking for structure

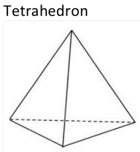
Ask students to draw or make a model of a structure that they can see.

For example, they could use matchsticks, modelling clay and polythene film to make a model of this diamond crystal structure.

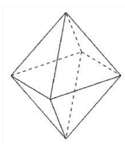
Diamond crystal in matrix

Look at this image of a diamond in its matrix rock.
What structure does it appear to have?

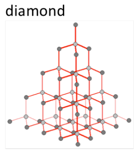
Tetrahedron



Octahedron



Carbon bonding in a diamond



ACTIVITY B: CLASSIFYING AND DEFINING

Time needed - 30 minutes

Classification and definition clearly play key roles in Science and Mathematics. Here we are not only concerned with learning classifications and definitions devised by others, but also with students engaging in these processes to gain an understanding of how Scientific and Mathematical concepts come about. In these activities, students examine a collection of 'objects' carefully, and classify them according to their different attributes. Students select each object, discriminate between that object and other similar objects (what is the same and what is different?) and create and use categories to build definitions. This type of activity is powerful in helping students understand what is meant by different terms and symbols, and the process through which they are developed.

- Work on some of the activities on **Handout 2**.
- What kinds of 'objects' do you ask students to classify and define in your classroom?
- Try to develop an activity using one of these types for use in your own classroom. Try to devise examples that force students to observe the properties of objects carefully, and that will create discussion about definitions.
- Try out your activity and report back on it in a later session.

The types of activity shown here may be extended to almost any context. In Mathematics, for example, the objects being decribed, defined and classified could be numerical, geometric or algebraic. In Science they could be organisms or elements. The activity here is for teachers to try to explore the range of possibilities.

Similarities and differences

In the examples shown, students may, for example, decide that the square is the odd one out because it has a different perimeter to the other shapes (which both have the same perimeter); the rectangle is the odd one out because it has a different area to the others and so on. Properties considered may include area, perimeter, symmetry, angle, convexity etc. In the silhouettes, students may consider many aspects: where the animals live, how they move, reproduce etc. Participants should try to devise their own examples.

Properties and definitions

None of the properties by themselves defines the square. It is interesting to consider what other shapes are included if just one property is taken. For example, when the property is "Two equal diagonals" then all rectangles and isosceles trapezia are included - but is that all the cases?
Taken two at a time, then results are not so obvious. For example, "Four equal sides" and "four right angles" defines a square, but "diagonals meet at right angles" and "four equal sides" does not (what else could this be?).

Creating and testing definitions

Participants usually write a rather vague definition of "polygon" or "bird" to begin with, such as: "A shape with straight edges" or an "animal that flies". They then see that this is inadequate for the given examples. This causes them to redefine more rigorously, like "a plane figure that is bounded by a closed path or circuit, composed of a finite sequence of straight line segments". Defining is a difficult area, and students should realise that there are competing definitions for the same idea (such as "dimension", for example).

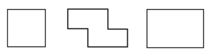
Classifying using two-way tables

Two-way tables are not the only representation that may be used, of course, and participants may suggest others. Venn diagrams and tree diagrams are just two examples used both in science and maths.

Handout 2: Classifying and defining

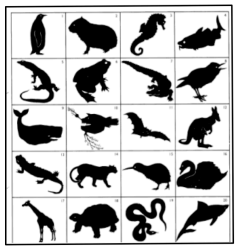
Similarities and differences

Show students three objects.
"Which is the odd one out?"
"Describe properties that two share that the third does not."
"Choose a different object from the three and justify it as the odd one out."



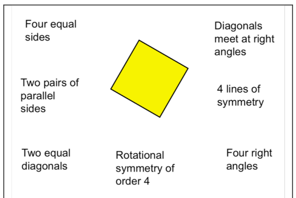
(a) $y = x^2 - 6x + 8$
(b) $y = x^2 - 6x + 9$
(c) $y = x^2 - 6x + 10$

Show students some silhouettes of animals.
"Can you name the animals?"
"Cut out the 20 cards and arrange the animals into groups."
"Write down the criteria you used to establish the groups."
"Show your groups to another student. Can they work out what your criteria were from your groupings?"

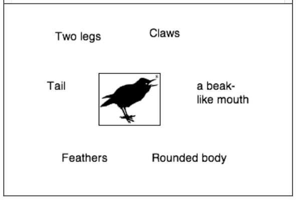


Properties and definitions

Show students an object.
"Look at this object and write down all its properties."
"Does any *single* property constitute a *definition* of the object? If not, what other object has that property?"
"Which *pairs* of properties constitute a definition and which pairs do not?"



"Look at this animal and write down all its features."
"Does any *single* feature uniquely identify the bird? If not, what other animal has that property/feature?"
"Which *pairs* of properties would uniquely decribe the bird? which pairs do not?"



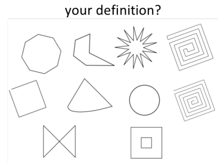
Creating and testing a definition

Ask students to write down the definition of a polygon, or some other mathematical word.

"Exchange definitions and try to improve them."

Show students a collection of objects.
"Use your definition to sort the objects."
"Now improve your definitions."

Which of these is a polygon according to your definition?

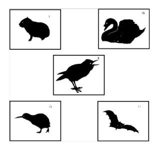


Ask students to write down a description of a bird, or some other plant or animal.

"Exchange descriptions and try to improve them."

Ask the students to look at silhouettes of some animals.
"Using only your description, decide which of these animals can be called 'birds'."
"Now improve your description."

Which of these is a bird according to your description?

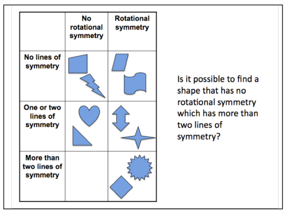


Classifying using a two-way table

Give students a two-way table to sort a collection of objects.

"Create your own objects and add these to the table."

"Try to justify why particular entries are impossible to fill."



(The silhouettes of animals are taken from Nuffield-Chelsea Curriculum Trust, 1987).

ACTIVITY C: REPRESENTING AND TRANSLATING

Time needed: 20 minutes

Mathematical and Scientific concepts have many representations; words, diagrams, algebraic symbols, tables, graphs and so on. It is important for students to learn to 'speak' these representations fluently and to learn to translate between them. It is helpful to think of each cell in the grid below as a translation process. Some translations are more common than others in classroom activities. For example, we often ask students to move between tables and graphs. This is labelled as 'plotting'.

from\to	**words**	**pictures**	**tables**	**graphs**	**formulae**
words					
pictures					
tables				plotting	
graphs					
formulae					

- Which representations do you use most often in your classroom?
- Which translation processes do you emphasise most? Which receive less attention?
- Discuss the examples shown in **Handout 3**.

As participants work on the activities they may begin to realise that some of these are less common in their classroom. Some notes on each activity are given below:

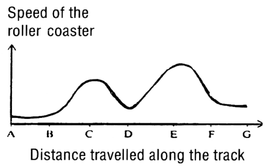
Job times

The words describe an inverse proportion, such as the following.

Number of people	1	2	3	4	5	6
Time taken in hours	24	12	8	6	4.8	4

Roller coaster

A suitable graph is shown below. It is interesting to note how difficult some students find this, particulaly when they misinterpret the graph as a picture of the situation.



Words and formulae

$$n \rightarrow \frac{2n+6}{2} - n = 3$$

Students enjoy trying to construct these and making them as difficult as possible!

Tables and graphs

This particular example focuses on graph sketching rather than graph plotting.

Tournament

$$m = n(n-1)$$

The diagram shows the structure of the situation. There are n^2 - n cells.

	A	B	C	D	E
A		AvB	AvC	AvD	AvE
B	BvA		BvC	BvD	BvE
C	CvA	CvB		CvD	CvE
D	DvA	DvB	DvC		DvE
E	EvA	EvB	EvC	EvD	

Penguins

The weight is proportional to volume, then dimensional analysis should suggest that weight is proportional to the cube of the height if the penguins are geometrically similar. This turns out to be a reasonable assumption and an approximate model is:

$$w = 20\,h^3$$

where h is the height in metres, and w is the weight in kg.

Handout 3: Translating between representations

Translating between representations

Words and tables

Given a verbal description, students are asked to produce a table of values.

Given a table, students are asked to describe the relationship in words.

Job times

Construct a table to show this relationship:

" If we double the number of people on the job, we will halve the time needed to complete it."

Number of people	1	2	3	4	5	6
Time taken in hours						

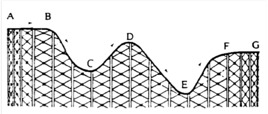
Pictures and graphs

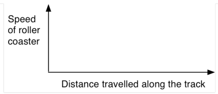
Given a picture of a situation, students imagine how the situation might evolve with time and sketch a graph

Given a graph, students are asked to sketch the corresponding picture of the situation

Roller coaster

Sketch a graph to show the speed of the roller coater as it travels along the track.





Words and formulae

Students are asked to symbolise a "think of a number" type problem,, and thus explain why it works.

Students invent an algebraic identity and then devise a "think of a number" problem to accompany it.

Think of number

"Think of a number. Double it. Add 6.
Divide by 2. subtract the number you first thought of.
Show that the answer is always 3."

Create your own example.

Tables and graphs

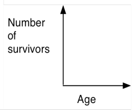
Students are asked to sketch a graph from a given table of data, without plotting.

Students devise a table of data that would fit a given sketch graph.

Life expectancy

Sketch a graph to fit the data

Age (yrs)	Number of survivors	Age (yrs)	Number of survivors
0	1000	50	913
5	979	60	808
10	978	70	579
20	972	80	248
30	963	90	32
40	950	100	1



Translating between representations (continued)

Tables and formulae

Given a table of data, students search for a general rule which governs it.

Students use this rule to make predictions.

Tournaments

The table shows the number of matches (*m*) that are required for a league tournament, where each team plays every other team twice, once at home and once away. Find a formula that gives the relationship between the number of teams (*n*) and the number of matches (*m*).

Number of teams (*n*)	2	3	4	5	6	7	8
Number of matches (*m*)	2	6	12	20	30	42	56

Use your formula to predict new entries in the table.
(E.g. How many matches do 20 teams require ?)

Formulae and graphs

Students plot the points on a spreadsheet and try to fit an algebraic function to the data using trial and improvement methods.

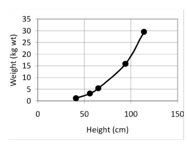
This involves translating directly back and forth between graphs and formulae, building up valuable intuitions about the shapes of various functions.

Penguins

Try to fit a function of the form $y = ax^n$ to the graph showing average heights and weights of five types of penguin.

Predict the weight of a now extinct penguin whose height was believed to be 150 cm.

	Height (cm)	Weight (kg wt)
Emperor	114	29.48
King	94	15.88
Yellow eyed	65	5.44
Fjordland	56	3.18
Southern blue	41	1.13



Roller coaster and *Life Expectancy* were taken from Swan (1985) *The Language of Functions and Graphs*, Shell Centre for Mathematical Education/Joint Matriculation Board. *Tournments* was adapted from Swan (1983) *Problems with Patterns and Numbers*, Shell Centre for Mathematical Education/Joint Matriculation Board. These examples also appeared in Swan and Crust (1993) *Mathematics Programmes of Study, Inset for Key Stages 3 and 4*, National Curriculum Council, York.

ACTIVITY D: MAKING CONNECTIONS

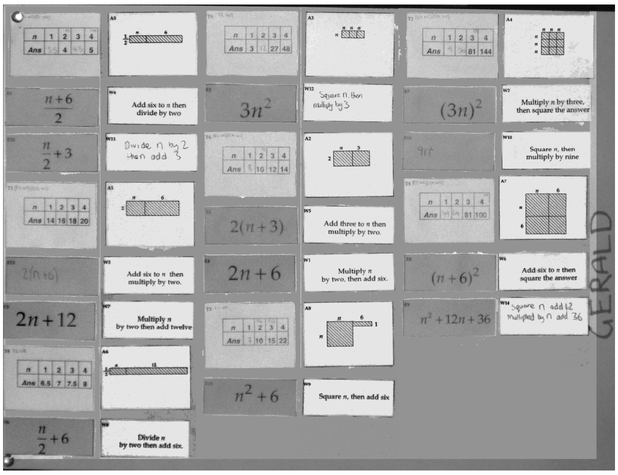
Time needed: 20 minutes

The activity in **Handout 4** is intended to encourage students to discuss connections between verbal, numeric, spatial and algebraic representations. For the following activity, participants should work in pairs or threes. They should begin by cutting out the cards.

- Cut out the set of cards on **Handout 4.**
- Take it in turns to match Card Set A: *algebra expressions* with the Card Set B: *verbal descriptions*. Place pairs of cards side-by-side, face up on the table. If you find cards are missing, create these for yourself.
- Next, match Card set C: *tables* to the cards that you have already matched. You may find that a table matches more than one algebra expression. How can you convince yourself or your students that this will always be true, whatever the value for *n*?
- Next, match Card set D: *areas* to those cards that have already been grouped together. How do these cards help you to explain why different algebra expressions are equivalent?
- Discuss the difficulties that your students would have with this task.

The final matching may be made into a poster, as has been done here.

The next activity encourages participants to compare their own thinking with an episode of learning from the classroom. The students on the 5 minute video clip are all low attaining 16-17 years old who have had very little understanding of algebra previously.



- Watch the **video clip.**
- What difficulties do the students have while working on this task?
- How is the teacher helping students?

Finally, participants may begin to consider how this type of activity may be applied to representations that they teach.

- Devise your own set of cards that will help your students translate between different representations that you are teaching.

Handout 4: Representing and making connections

Each group of students is given a set of cards. They are invited to sort the cards into sets, so that each set of cards have equivalent meaning. As they do this, they have to explain how they know that cards are equivalent. They also construct for themselves any cards that are missing. The cards are designed to force students to discriminate between commonly confused representations.

Card Set A: Algebra expressions

E1 $\frac{n+6}{2}$	E2 $3n^2$
E3 $2n+12$	E4 $2n+6$
E5 $2(n+3)$	E6 $\frac{n}{2}+6$
E7 $(3n)^2$	E8 $(n+6)^2$
E9 $n^2+12n+36$	E10 $3+\frac{n}{2}$
E11 n^2+6	E12 n^2+6^2
E13	E14

Card Set B: Verbal descriptions

W1 Multiply n by two, then add six.	W2 Multiply n by three, then square the answer
W3 Add six to n then multiply by two.	W4 Add six to n then divide by two
W5 Add three to n then multiply by two.	W6 Add six to n then square the answer
W7 Multiply n by two then add twelve	W8 Divide n by two then add six.
W9 Square n, then add six	W10 Square n, then multiply by nine
W11	W12
W13	W14

Card Set C: Tables

T1

n	1	2	3	4
Ans	14	16	18	20

T2

n	1	2	3	4
Ans			81	144

T3

n	1	2	3	4
Ans		10	15	22

T4

n	1	2	3	4
Ans	3		27	48

T5

n	1	2	3	4
Ans			81	100

T6

n	1	2	3	4
Ans		10	12	14

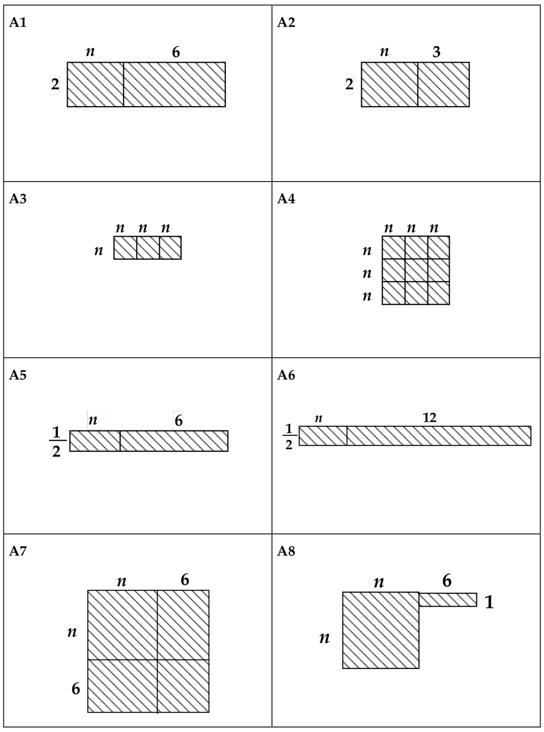
T7

n	1	2	3	4
Ans		4		5

T8

n	1	2	3	4
Ans	6.5	7	7.5	8

Card Set D: Areas



Swan, M. (2008), *A Designer Speaks: Designing a Multiple Representation Learning Experience in Secondary Algebra*. Educational Designer: Journal of the International Society for Design and Development in Education, 1(1), article 3.

ACTIVITY E: ESTIMATING

Time needed: 20 minutes.

Estimation problems involve students making assumptions, then working with these assumptions to build chains of reasoning. It is often the case that, individually, students feel unable to cope with such problems but when they work collaboratively, they are surprised at how much knowledge they can build on.

- In pairs or small groups, work together on the trees problem on **Handout 5**.
- When each group has produced a reasoned answer, take it in turns to explain your solutions, describing all the assumptions you have made.
- In which solution do you have most confidence? Why is this?

The following shows just one approach that teachers have adopted:

1. Estimate the number of teachers in the country.
2. Estimate the size of the average family.
3. Estimate the volume of a typical newspaper.
4. Assuming that each family buys one newspaper per day, estimate the total volume of newspaper consumed per day.
5. Estimate the radius and height of the useable part of a suitable tree.
6. Calculate the volume of the trunk.
7. Assuming that the total volume of the trunk is converted into newsprint, use your answers from (4) and (6) to estimate the required number of trees.

The following data was supplied by the forestry commission, and may provide a useful independent check:

> "The example assumes that the whole tree goes for paper. In reality only the smaller end would be used. Approximately 2.8kg of wood will make 1 kg of newsprint. 1 cubic metre of wood, freshly felled, as supplied to a pulp mill, weights about 920 kg. This is based on the Sitka spruce and is the average throughout the year. At the time of felling at the age of 55 years, each tree will have a volume of 0.6 cubic metres, including the bark. The diameter at 1.4 metres from the ground would be 27 cm."

- Make a list of estimation problems that would be accessible to one of your classes.
- Discuss how you might organise a lesson based on an estimation problem.

A possible list of questions might be:

- How much do you drink in one year?
- How many teachers are there in your country?
- How long would it take you to read out all the numbers from one to one million? Would this be different in different languages?
- How many people could stand comfortably in your classroom?
- How many times does a person's hear beat in one year?
- How many exercise books do you fill in your school career?
- How many pet dogs are there in your town?

Handout 5 Estimating

Work on the following problem together.

Trees

About how many trees are needed each day to provide newspapers for your country?

Try to make a reasonable estimate based on facts that you already know.

In solving this question, you have had to make assumptions and construct a chain of reasoning.

Write down a list of estimation questions that would be suitable for your own class.

ACTIVITY F: MEASURING AND QUANTIFYING

Time needed: 20 minutes.

Our society creates and uses measures all the time. We create measures for fundamental concepts (e.g. length, time, mass, gradient, speed, density) and more complex social constructs (e.g. academic ability, wealth, inflation, job performance, quality of education, sporting prowess, physical beauty). Scientists and Mathematicians devise measures in order to seek patterns, relationships and laws. Politicians use measures to monitor and control. All educated citizens should realise that many of these measures are open to criticism and improvement.

- What kinds of measure do you meet in your everyday life? Make a list on **Handout 6**.
- What kinds of measure do your students experience?

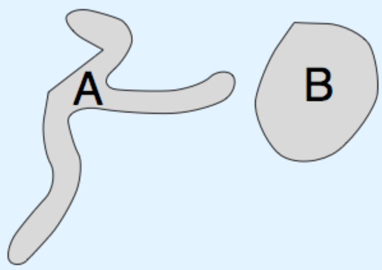
On **Handout 6**, two types of activity are suggested for students.

- Work on the *measuring slope* task together.
- Try to arrive at a convincing explanation as to why *height of step ÷ length of step* is the better measure for slope.
- Can you think of other examples of alternative measures for the same concept?

The ratio *height of step ÷ length of step* is better than the difference *height of step - length of step* because the ratio is dimensionless. This means that if you geometrically enlarge a staircase, the ratio will not change, whereas the difference will.

The final activity suggests devising a measure for an everyday phenomenon. Participants may like to start this by thinking about "compactness":

Over recent years, geographers have tried to find ways of defining the shape of an area of land. In particular, they have tried to devise a measure of 'compactness'. You probably have some intuitive idea of what 'compact' means already. On the right are two islands. Island B is more compact than island A. 'Compactness' has nothing to do with the size of the island. You can have small, compact islands and large compact islands.



- Draw some shapes and put them in order of compactness.
- Try to agree what is meant by the term.
- Is area ÷ perimeter a good measure of compactness? Why or why not?
- Try to devise several ways of measuring compactness. Try to make your measures range from 0 to 1, where 1 is given to a shape that is perfectly compact.
- Afterwards, compare your ideas with those used by geographers on **Handout 7.**
- Finally, consider other everyday phenomena and consider how you would measure them (return to Handout 6).

Handouts 6 and 7 Measuring and quantifying

What measures do your students meet in everyday life?

Make a list:

Possible activities for students:

Comparing measures

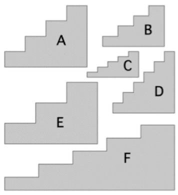
Give students two ways of measuring something. Ask students to compare them and say why one is better than another.

Measuring slope

Put these staircases in order of steepness.

Is **"Height of step - length of step"** a good measure of steepness?

Why is **"Height of step ÷ length of step"** better?



Creating measures

Ask students to devise a measure for an everyday phenomenon and then use it.

How would you measure:

- the "compactness" of a geometrical shape?
- the "stickiness" of adhesive tape?
- the "bendiness" of a river?
- the "difficulty" of a bend in the road?
- the "fitness" of a person?

Measuring compactness

The inadequacy of using *area ÷ perimeter* as a measure of compactness may be seen by comparing two similar shapes of different sizes. Consider, say a square of side two units and a square of side three units. We would say that they are equally compact as they are both squares, but using the ratio *area ÷ perimeter*, their measures would be different: 4/8 = 0.5 and 9/12 = 0.75.

We could adapt this measure to make it dimensionless by using the formula: $C = \frac{a}{p^2}$,

where a = area and p = perimeter. This would then give the value 1/16 to both squares. This ratio takes a maximum value when the shape is circular. In this case, $C = \frac{\pi r^2}{(2\pi r)^2} = \frac{1}{4\pi}$.

In order to make the measure lie between 0 and 1, we could therefore scale the measure by multiplying by 4π. This is used by geographers and is called the **Circularity ratio** (Selkirk, 1982):

Circularity ratio

$$C_1 = \frac{4\pi a}{p^2}$$ where a = area; p = perimeter of the shape

One criticism of this measure is that it is difficult to define and calculate p when one is trying to measure very large, irregular boundaries like countries or river basins. Other possible measures, also quoted by Selkirk, are:

Form ratio

$$C_2 = \frac{4a}{\pi l^2}$$ where a = area; l = length of a line joining the two most distant points

Compactness ratio

$$C_3 = \frac{a}{\pi R^2}$$ where a = area; R = radius of smallest circle that surrounds the shape

Radius ratio

$$C_4 = \frac{r}{R}$$ where r = radius of largest circle that will fit inside the shape;

R = radius of smallest circle that surrounds the shape

Reference: Selkirk, K (1982) *Pattern and Place - An Introduction to the Mathematics of Geography*, Cambridge University Press.

ACTIVITY G: EVALUATING STATEMENTS, RESULTS AND REASONING

Students that are actively learning are constantly challenging hypotheses and conjectures made by others. The activities considered here are all designed to encourage this kind of behaviour.

Ask participants to work together in groups of two or three using the activity of **Handout 7.**

In this activity, you are given a collection of statements.

- Decide on the validity of each statement and give explanations for your decisions. Your explanations will involve generating examples and counterexamples to support or refute the statements.
- In addition, you may be able to add conditions or otherwise revise the statements so that they become 'always true'.
- Create some statements that will create a stimulating discussion in your classroom.

This kind of activity is very powerful. The statements may be prepared to encourage students to confront and discuss common misconceptions or errors. The role of the teacher is to prompt students to offer justifications, examples, counterexamples. For example:

Pay rise:
"OK you think it is sometimes true, depending on what Max and Jim earn. Can you give me a case where Jim gets the bigger pay rise? Can you give me an example where they both get the same pay rise?"

Area and perimeter:
"Can you give me an example of a cut that would make the perimeter bigger and the area smaller?"
"Suppose I take a bite out of this triangular sandwich. What happens to its area and perimeter?"

Right angles.
Can you *prove* this is always true?

Bigger fractions
You think this is always true? Can you draw me a diagram to convince me that this is so?
What happens when you start with a fraction greater than one?

Handout 7: Always, sometimes or never true?

Pay rise Max gets a pay rise of 30%. Jim gets a pay rise of 25%. So Max gets the bigger pay rise.	**Sale** In a sale, every price was reduced by 25%. After the sale every price was increased by 25%. So prices went back to where they started.
Area and perimeter When you cut a piece off a shape you reduce its area and perimeter.	**Right angles** A pentagon has fewer right angles than a rectangle.
Birthdays In a class of ten students, the probability of two students being born on the same day of the week is one.	**Lottery** In a lottery, the six numbers 3, 12, 26, 37, 44, 45 are more likely to come up than the six numbers 1, 2, 3, 4, 5, 6.
Bigger fractions If you add the same number to the top and bottom of a fraction, the fraction gets bigger in value.	**Smaller fractions** If you divide the top and bottom of a fraction by the same number, the fraction gets smaller in value.
Square roots The square root of a number is less than or equal to the number	**Series** If the limit of the sequence of terms in an infinite series is zero, then the sum of the series is zero.

ACTIVITY H: EXPERIMENTING AND CONTROLLING VARIABLES

Time needed: 40 minutes.

Two activities are presented here. One involves the planning of an experiment, the other involves a computer applet that is presented with this unit.

Start by discussing the first two situations on **Handout 8.**

- Choose one of the scientific questions shown in *Devising a fair test.*
- Work on the experimental design in a small group.
- Often in science classrooms, the teacher designs the experiments and students carry them out. Handing over the experimental design decisions presents many challenges for both teachers and students. For example, students may ask for equipment that you do not have readily available. What other challenges are there? Make a list.

Now ask participants to consider the final problem *Body Mass index*.

- Work on the Body Mass Index problem in pairs, using the computer applet.
- Note down the method you adopt.
- Now watch the **video clip** showing a lesson with students.
    - How did the teacher organise the lesson? What phases did it go through?
    - Why do you think she organised it this way?
    - How did the teacher introduce the problem to students?
    - What different approaches were being used by students?
    - How did the teacher support the students that were struggling?
    - How did the teacher encourage the sharing of approaches and strategies?
    - What do you think these students were learning?

It is easy to find the boundaries at which someone becomes underweight/overweight/obese if one variable is held constant while the other is varied systematically. The boundaries occur at:

	BMI
Underweight	Below 18.5
Ideal weight	18.5 - 24.9
Overweight	25.0 - 29.9
Obesity	30.0 and Above

In order to find out how the calculator works, it is better to forget realistic values for height and weight and simply hold one variable constant while changing the other systematically. For example, if students hold the height constant at 2 metres (not worrying if this is realistic!), then they will obtain the following table and/or graph:

Weight (kg)	60	70	80	90	100	110	120	130
BMI	15	17.5	20	22.5	25	27.5	30	32.5
	Underweight		Ideal weight		Overweight		Obese	

From this it can be seen that there is a proportional relationship between weight and BMI. (If you double weight, you double BMI; Here BMI = Weight/4)

Handout 8: Experimenting and controlling variables

Devising a fair test

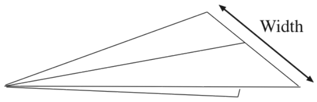
Students are asked to devise and conduct an experiment to find the relationship between two or more variables. As they do this, they must consider how they will control other variables.

As they do this, they must consider how they will control other variables.

One lump or two?

It takes some time for sugar cubes to dissolve in coffee. What factors might affect the rate of dissolving? Devise and conduct an experiment to investigate the relationship between the rate of dissolving and one of these factors.

Paper aeroplane



Alice wants to know how to make a paper aeroplane that will fly for a long time. What factors might affect the flight time?

Devise and conduct an experiment to investigate the relationship between the flight time and one of these factors.

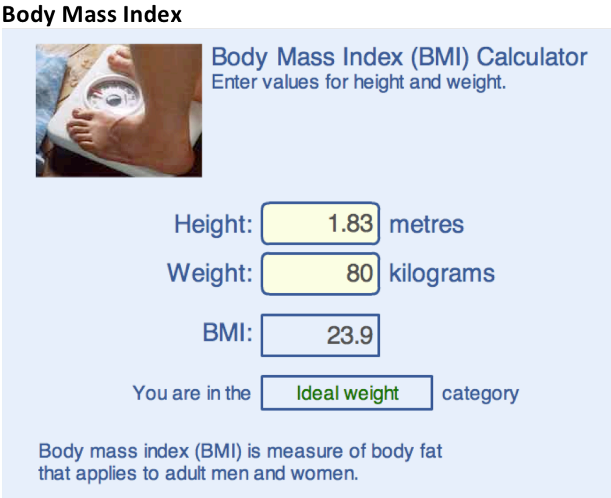
Exploring how a calculator works

Students are given a spreadsheet or online calculator to explore. The challenge is to find out how it works.

For example, the calculator shown here is used on websites to help an adult decide if he or she is overweight. Students enter values for heights and weights and collect data in order to discover how the calculator calculates the BMI.

There are many other examples online.

Body Mass Index



The BMI activity is taken from Swan, M; Pead, D (2008). *Professional development resources*. Bowland Maths Key Stage 3, Bowland Trust/ Department for Children, Schools and Families. Available online in the UK at: http://www.bowlandmaths.org.uk. It is used here by permission of the Bowland Trust.

ACTIVITY I: PLAN A LESSON, TEACH IT AND REFLECT ON THE OUTCOMES

Time needed:

- ***15 minutes discussion before the lesson***
- ***1 hour for the lesson***
- ***15 minutes after the lesson***

Choose one of the problems in this unit that you feel would be appropriate for your class.

Discuss how you will:

- Organise the classroom and the resources needed.
- Introduce the problem to students.
- Explain to students how you want them to work together.
- Challenge/assist students that find the problem straightforward/ difficult.
- Help them share and learn from alternative problem-solving strategies.
- Conclude the lesson.

If you are working on this module with a group, it will be helpful if each participant chooses the same problem, as this will facilitate the follow-up discussion.

Now you have taught the lesson, it is time to reflect on what happened.

- What range of responses did students have to the task?
  Did some appear confident? Did some need help? What sort of help? Why did they need it?
- What different scientific processes did students use?
  Share two or three different examples of students' work.
- What support and guidance did you feel obliged to give?
  Why was this? Did you give too much or too little help?
- What do you think students learned from this lesson?

FURTHER READING

Swan, M (2005) Improving Learning in Mathematics: Challenges and Strategies, Department for Education and skills and dowloadable from:
http://www.nationalstemcentre.org.uk/elibrary/resource/1015/improving-learning-in-mathematics-challenges-and-strategies

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Wood, D. (1988). *How Children Think and Learn*. Oxford and Cambridge, MA: Blackwell.
Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of child psychology and psychiatry, 17*, 89-100.