IncluSMe
Intercultural learning in Science and Mathematics initial teacher education

Module 8

THE RELEVANCE OF LANGUAGE FOR MATHEMATICS EDUCATION

Worksheets

Co-funded by the Erasmus+ Programme of the European Union

This *worksheet* is based on the work within the project Intercultural learning in mathematics and science initial teacher education (IncluSMe). Coordination: Prof. Dr. Katja Maaß, International Centre for STEM Education (ICSE) at the University of Education Freiburg, Germany. Partners: University of Nicosia, Cyprus; University of Hradec Králové, Czech Republic; University of Jaen, Spain; National and Kapodistrian University of Athens, Greece; Vilnius University, Lithuania; University of Malta, Malta; Utrecht University, Netherlands; Norwegian University of Science and Technology, Norway; Jönköping University, Sweden; Constantine the Philosopher University, Slovakia.

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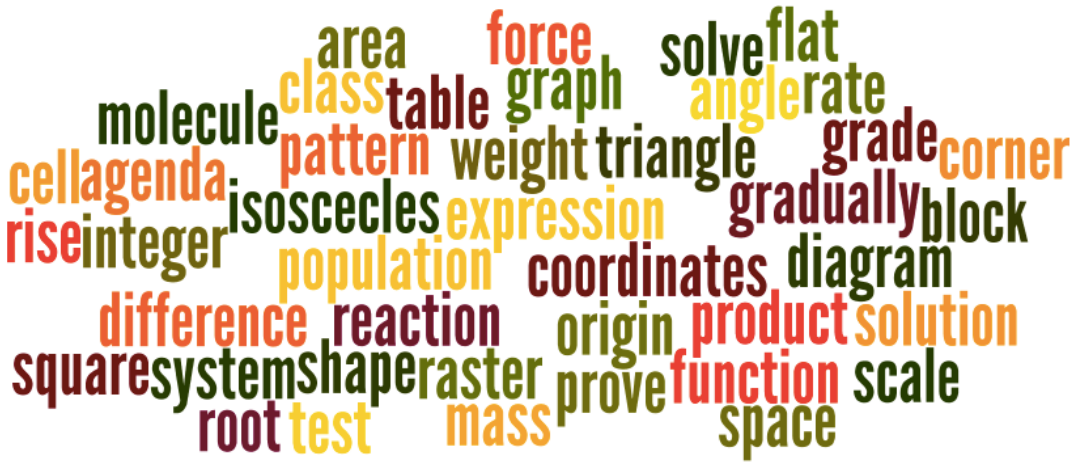
I. Introduction into the topic: The relevance of language in mathematics

Activity 1.1: Types of language

Duration: 30 minutes

Look at the words below.



- What do you notice? What kinds of words do you see?
- Individually select about 5 words that are typical for your subject (mathematics).
- Share these in a small group and discuss the questions below:
    - Which of these words are also part of our daily language? What is the difference in meaning in mathematics compared to in daily life? What is a typical expressions (or sentence) using this word in daily language and what is a typical expression in mathematics?
    - What problems do you expect your students may have regarding this 'shift of' meaning? Is this different for students who are native speakers in your language and for students who are not?
- In the whole group share the problems that your small group identified.

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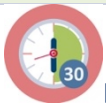
I. Introduction into the topic: The relevance of language in mathematics

Activity 1.2: Student difficulties



Duration 30 minutes

Part 1

- Study the mathematical problem below and solve it

In a region there are 4 primary schools. The number of pupils in 1996 and 1997 for each school is presented in the table below.

Number of students	***School 1***	***School 2***	***School 3***	***School 4***
In 1996	338	182	220	203
In 1997	237	160	270	227

For which school is the increase in pupils the highest?

- Discuss in a small group what problems students (grade 6 or 7) may have when solving this problem? Are some of these problems language related?

Part 2

- Study the excerpt below. It is a dialogue between a Student and a Teacher-researcher. The student has read the problem above aloud, solved it and now explains what he did.

> *S: For which school, for which school the increase is the highest? Then you need to look…. School1, in 1996 and 1997, in 1996 they had 338 and 237, this you add together, then you go to school 2….*
> *T: Why do you choose to add?*
> *S: Because they ask for which school the increase is the highest, so I have to for those 4 schools, you have to add the number of pupils in 1996 and in 1997, those you have to add together and then, you see, if you have those four results, then you look which number is highest and let's say.. School 1 is highest, this you write here.*
> *[…]*
> *T: and what, for example, does increase mean?*
> *S: Eh, for which school … eh …where is it highest, let's say, where is it …. actually ….[not clear] … of which … [not clear] schools…*
> *T: But that, how do you do this? You read it and then you think 'it must be this and ….*

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S: yes…
T: or do you say 'no, I'm sure this is meant by increase'.
S: sometimes I forget. But for this problem it is easy, because you already hear 'highest', then you know. Then you do not really need to look at increase. Then you have to see for which school it is highest. So actually you can leave out increase and then you can just calculate it.
T: and if you leave it out, then it says 'for which school the highest'?'
S: Yes
T: The highest. And then?
S: Yes, but then you actually don't know what it is about. Yes. Here it is about pupils. Eh, the highest.
[….]
T: but here for example, increase is an important word.
S: ….
T: isn't it?
S: yes, here it is.
T: Here it is.
S: yes, but even so I understand it. I got it.

- Discuss the following questions in your small group.
-
    - How do you value the students' language abilities in general?
    - What strategies does the student use to overcome language difficulties?
    - What do you notice about the student –teacher/researcher interaction?
    - Do you have similar experiences? What would you do in such a situation?
    - How would you provide language support for this student?

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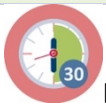
II. Background: language and mathematics education

Activity 2.2: Analysing a textbook problem



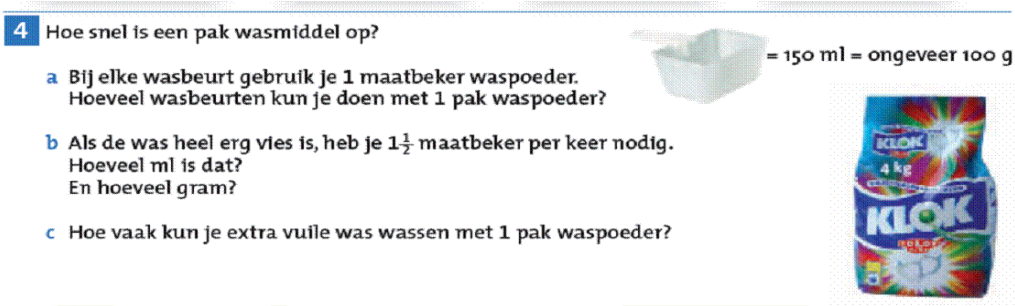
Duration: 30 minutes

The problem below is taken from a Dutch mathematics textbook for pupils in grade 6 or 7 (age 11-12). You see the original Dutch problem with the illustrations and a translation of the text in English.

Study the problem and answer and discuss the questions below:

Washing-powder

a. With every washing you use 1 measuring cup of washing-powder. How many washings can you do with 1 pack of washing-powder?
b. If the clothing is very dirty you need 1 ½ cup per wash.
   How many ml is that?
   And how many grams?
c. How often can you clean extra filthy laundry with 1 pack of washing-powder?



Translation of significant text in the illustrations.

- on the illustration with the measuring cup you see: *= 150 ml = about 100 g*.
- on the pack detergent you can read: 4 kg

Questions for the analysis of the problem

- What kind of language is used in the problem of the washing powder? Identify: daily language, mathematical language (words, symbols, …) and general academic language.
- Study the problem from a pupils' perspective: what might be difficult for the pupil?
- Think of ways to make the language more accessible to the pupil.

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II. Background: language and mathematics education

Activity 2.3: The role of the teacher scaffolding language

Duration 30 minutes

Part 1

Scaffolding language is: *Adaptive linguistic support that helps students in developing the language abilities that enables students to think and communicate independently in school subjects.* (see Gibbons, 2002, 2009)

Read the excerpt below and discuss in pairs the strategies the teachers (T) uses to scaffold students (S1-3) language.

Fragment 1	
S1	The sunflower grows gradually (points to the first segment of the graph). Is this like mathematical language? (Teacher nods.) Rises gradually. And then here (points to the second segment of the graph), then it grows fast. Yes, it grows fast. And that was it.
T	All right. And you can tell from the graph because…?
S1	Because it, that line, it just shows that it grows very fast.
T	And that, what would we call that in mathematical language?
S2	Rise.
	It rises quickly. OK. S3, next two ones.
S3	Here you can see that it changes direction again (holds his fingers at a segment on the graph), it goes little more to the right. And then it rises again to thirteen (moves his finger along the line graph until he meets the next segment). And then, it rises gradually up to constant.
T	It changes direction. And then what would you say, 'it'? What would you say about this (points to the third segment)?
S3	Then it goes gradually. Rising.
T	The graph rises.
S3	Up to thirteen.
T	Yes, the graph rises gradually.
S3	Gradually.
T	Gradually. And then what happens to the sunflower?
S3	Then it just stays like this (Abdul moves his hands in horizontal direction).

Source: Smit, J., Bakker, A., Van Eerde, D., & Kuijpers, M. (2006). Using genre pedagogy to promote student proficiency in the language required for interpreting line graphs. *Mathematics Education Research Journal, 28*, 457-478. doi:10.1007/s13394-016-0174-2

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Part 2

Explore the strategies for scaffolding language in the table below.
For each strategy think of another example that fits a topic you are teaching now. Also discuss in a small group which of these strategies you use and which you want to use.

Table 1. Strategies for scaffolding language and examples for each strategy

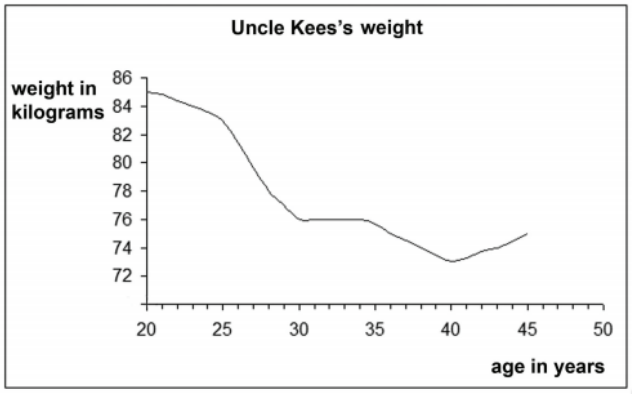
1	Reformulate pupils' utterances (spoken or written) into more academic wording	[In response to *the graph goes higher and higher up*:] *Indeed, the graph rises steeply.*
2	Ask pupils to be more precise in spoken language or to improve their spoken language	*What do you mean by 'it'?*
3	Repeat correct pupil utterances	*The graph descends slowly indeed.*
4	Refer to features of the text type (interpretative description of a line graph)	*Into how many segments can we split the graph?*
5	Use gestures or drawings to support verbal reasoning	For example, gesturing a horizontal axis when discussing this concept
6	Remind pupils (by gesturing or verbally) to use a designed scaffold (i.e., word list or writing plan) as a supporting material	*Look, the word you are looking for is written down for you here.*
7	Ask pupils how written text can be produced or improved	*How can we rewrite this in more mathematical wording?*

Source: Smit, J., van Eerde, H.A.A. & Bakker, A. (2013). A conceptualisation of whole-class scaffolding. *British Educational Research Journal*, 39 (5), (pp. 817-834) (18 p.).

Part 3

Read the two fragments below of teacher-student interactions and individually score the teachers utterances according to the strategies from the table above. Do the same for fragment 1 in part 1 of this worksheet. Compare and discuss your results in pairs.

The interactions are about this graph



Fragment 2

T: How can he tell his age? Where does he see that? S1.
S1: Down below.
T: Can you speak up? We cannot hear that.
S1: Down below, at (…?).
T: And down below, for that we know proper mathematical language. S2?
S2 Horizontal axis.
T: Why don't you put that in a sentence. It's just a word on its own like this.
S2: Along the horizontal axis it says age in years.
T: That's a beautiful sentence, isn't it? OK.

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Fragment 3

S1	When he was thirty and thirty-five, he just stayed at seventy-six kilograms (points at numbers along the axis).
T	Wait a minute. How can we formulate this in the proper wording? When he was thirty *and* thirty-five? What you, S2?
S2	Thirty till thirty- five, he just stayed the same.
T	Do we say thirty *till* thirty-five?
S3	From.
T	From. From, and now put this in a beautiful sentence. Just face the class, because then this lovely sentence will come out!
S1	From thirty till thirty-five Uncle Kees stays constant.
T	From thirty till thirty-five. When Uncle Kees is *between* thirty *and* thirty-five years old, or… from thirty till thirty-five.
S1	Uncle stays, uncle stays constant, uncle Kees stays constant.
T	Yes. But do I keep standing "constant"? What remains constant? Remember, I'm uncle Kees again (acts out uncle Kees).
S1	The kilograms.
T	And how, what was that again? Not my kilograms.
S1	Uncle Kees's weight just stays at seventy-six kilograms.
T	Yes. His weight remains the same. As you can tell from the graph?
S1	Which remains constant.
T	That's it. Now you have used proper mathematical language. All right. In a few minutes' time we've actually come to know a lot about Uncle Kees, who we haven't even met. By looking at the graph's change in direction. OK.

Source: Smit, J., Bakker, A., Van Eerde, D., & Kuijpers, M. (2006). Using genre pedagogy to promote student proficiency in the language required for interpreting line graphs. *Mathematics Education Research Journal, 28*, 457-478. doi:10.1007/s13394-016-0174-2

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III. Connecting to practice

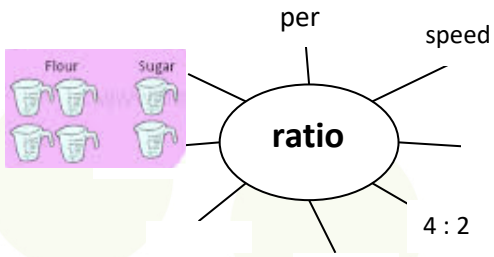
Activity 3.1: Tools for supporting language

Duration 30 minutes

Study the eight tools and activities below for stimulating language proficiency in mathematics. For each of these:

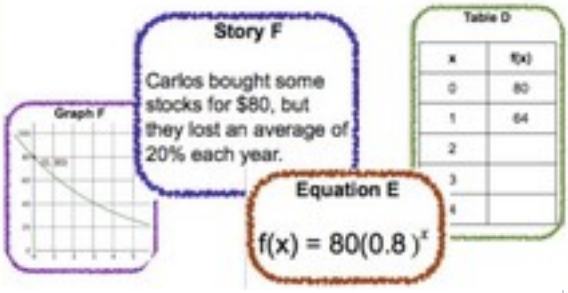
- Indicate how it contributes to the development of language proficiency and to the development of mathematical understanding. Try to generalize from the specific example to a more general use of the tool/activity.
- Formulate a subject-specific language goal you can address with this activity.
- Discuss your findings in a small group.

1. Asks students to make a mind-map for a concept



2. Provide visual models and different representations for better understanding

Chocolate bar divided by friends:	My Picture
1 bar - 2 friends	
1 bar - 3 friends	
1 bar - 4 friends	
1 bar - 5 friends	



Also encourage students to make this kind of drawings, schemes, graphs etc. themselves.

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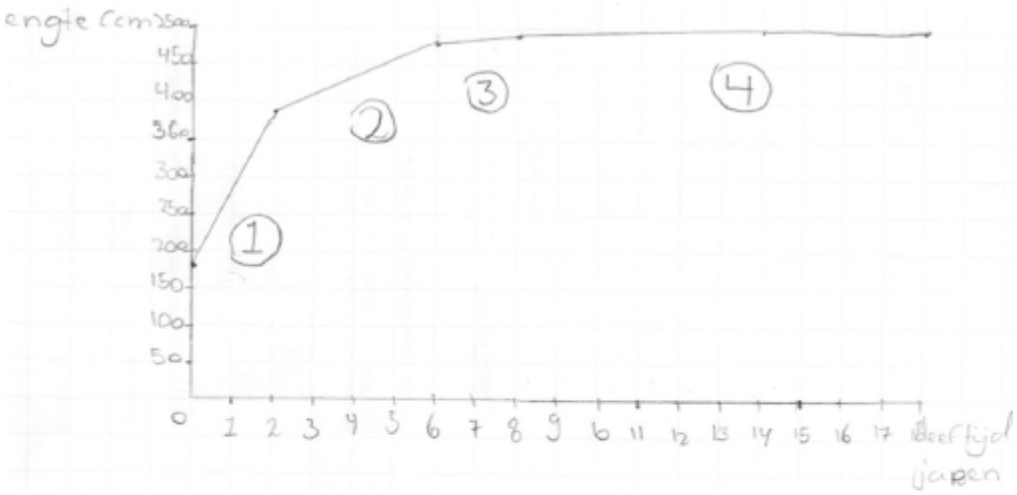
3. Present a writing-frame to help students write (example for a graph)



When Niek is born, he is .....

In his first years.....

You can tell as the graph.....

From the age of 2 onward .....

The graph .....

After his sixth birthday.....

You can see .....

When Niek is about 9 years old .....

The graph.....

In the end Niek .....

4. Have students make vocabulary lists – dictionary

- Students make their own vocabulary list with new words for a certain chapter. For each word they write a definition or explanation, formulate a typical sentence/expression, write related terms and symbols (e.g. percent-percentage-%) give an example, make a drawing etc.
- Have students exchange their lists and give feedback.

5. Organize peer-review on written work (solutions, definitions etc.)

Have students write an explanation or solution and hand in their work. Redistribute these to students in class and have them comment on the quality of the written texts (peer feedback).

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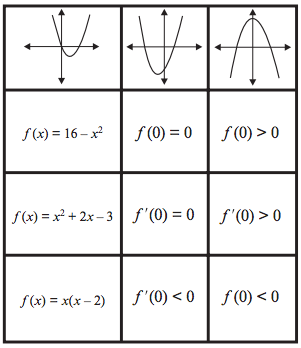
6. Use games combined with collaborative learning

These activities require students to talk about mathematical concepts

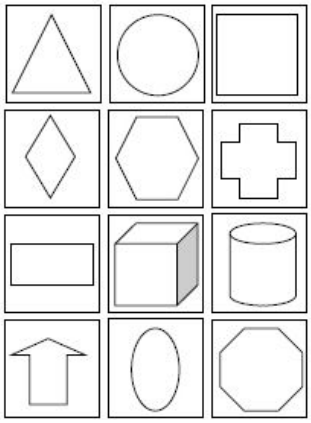
Games using cards, for example:

- matching text & representations;
- matching daily language-mathematical language
- classifying mathematical objects



Guessing games, for example:

- Guess what? (Based on Guess who?). Instead of names/persons you use shapes, graphs, formulas or other mathematical objects.
- Describe a shape (2D or 3D) or a graph (using mathematical terms) without using its name.
- Pictionary with mathematical words: a student present clues in a drawing another student guesses



7. Evaluating statements and reasoning

This form can be applied to any topic

'*Are these expression always-sometimes-never true?*

$p \div 12 = s + 12$	$3 + 2y = 5y$
$2t - 3 = 3 - 2t$	$q + 2 = q + 16$

8. Have student create problems

- Ask students to create problems/questions fitting the topic of the lesson.
- Ask students to create variants of questions from textbook or tests
- Ask students to write questions/problems that lead to a specific answer. For example: make three different equations for which is x=3 is a solution; drawthree different triangles that each have an area of 8 cm^2; etc.

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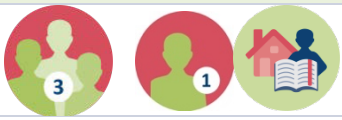
III. Connecting to practice

Activity 3.2 Analyse a textbook and design a lesson



Duration 60 mins

Part 1 - analysing

Select a paragraph form a mathematics textbook used in your (future) classes.

In pairs analyse this paragraph and identify language 'problems' your students may meet. Guiding question to use are:

- What types of language are being used? What are typical mathematical words and expressions? What about general academic language and daily language?
- Where do you expect problems to occur? Is this different for different types of students? Distinguish: native speakers and second language learners.
- Is attention paid to the meaning of words/concepts/expression? Are formulations clear? Are schemes, representations and visuals used, are they used in a clear way? If not: what problems do you foresee?
- Do the activities provide opportunities for language production (speaking and writing)?

Next think of ways to make the language more accessible to your students: what language goals can you formulate? How would you provide language support (scaffolding)? What tools or activities (see activity 3.1) can you use to make the lesson more language-sensitive?

Part 2 – (re)designing

Based on your analysis (re)design a lesson fitting the paragraph you analysed. You will:

- Formulate one or two content-related (mathematical) language goals for this lesson.
- Design one or two activities providing 'language support' to your students in reaching these goals (see activity 3.1).
- Justify your choices and decisions based on what you learned about language sensitive mathematics teaching.

Prepare a 1-2 minute pitch about your plans for the lesson/activity, to present at the end of this session. Be sure to address the way you support language development of your students.

Finish the design as homework, include goals, teaching materials and student materials. If possible, try it out in your class. After you used it in class fill out the evaluation form on worksheet 3.3.

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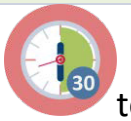
III. Connecting to practice

Activity 3.3: Sharing experiences and feedback



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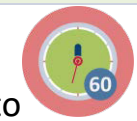

Duration: 30-60 minutes

Fill in the evaluation form below about your design and your teaching experiences
Prepare a presentation about these, reflecting on:

- What are your content-related mathematical language goals?
- What activities did you have students do? Why these?
- What was your role as a teacher?
- What are your experiences: successes and challenges?
-

Name			
School			
Topic		**grade**	
Which activity did you (re)design? Give a brief description of resources, characteristics of the (language-related) goals, the student activities and the teaching method(s).			
How does your activity /lesson contribute to the language development in connection to mathematical understanding? What tools did you include?			
How and why do you think that your lesson/activity connects to your students' language and mathematical abilities?			
Experiences during the try out: what student behavior did you observe (different than normal)? How did your students react to the activities? What did you observe with respect to language development?			

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