

IS IT MATHEMATICS?

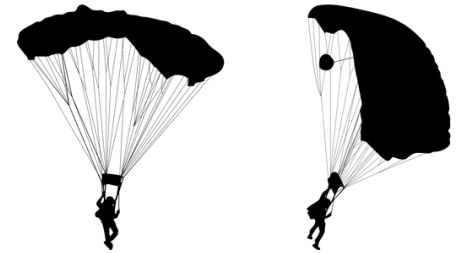
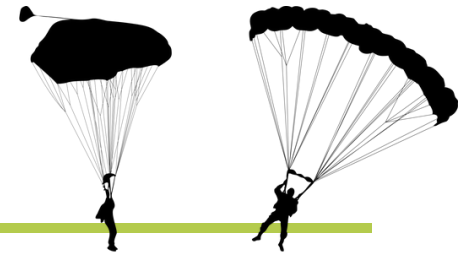
Modelling with functions

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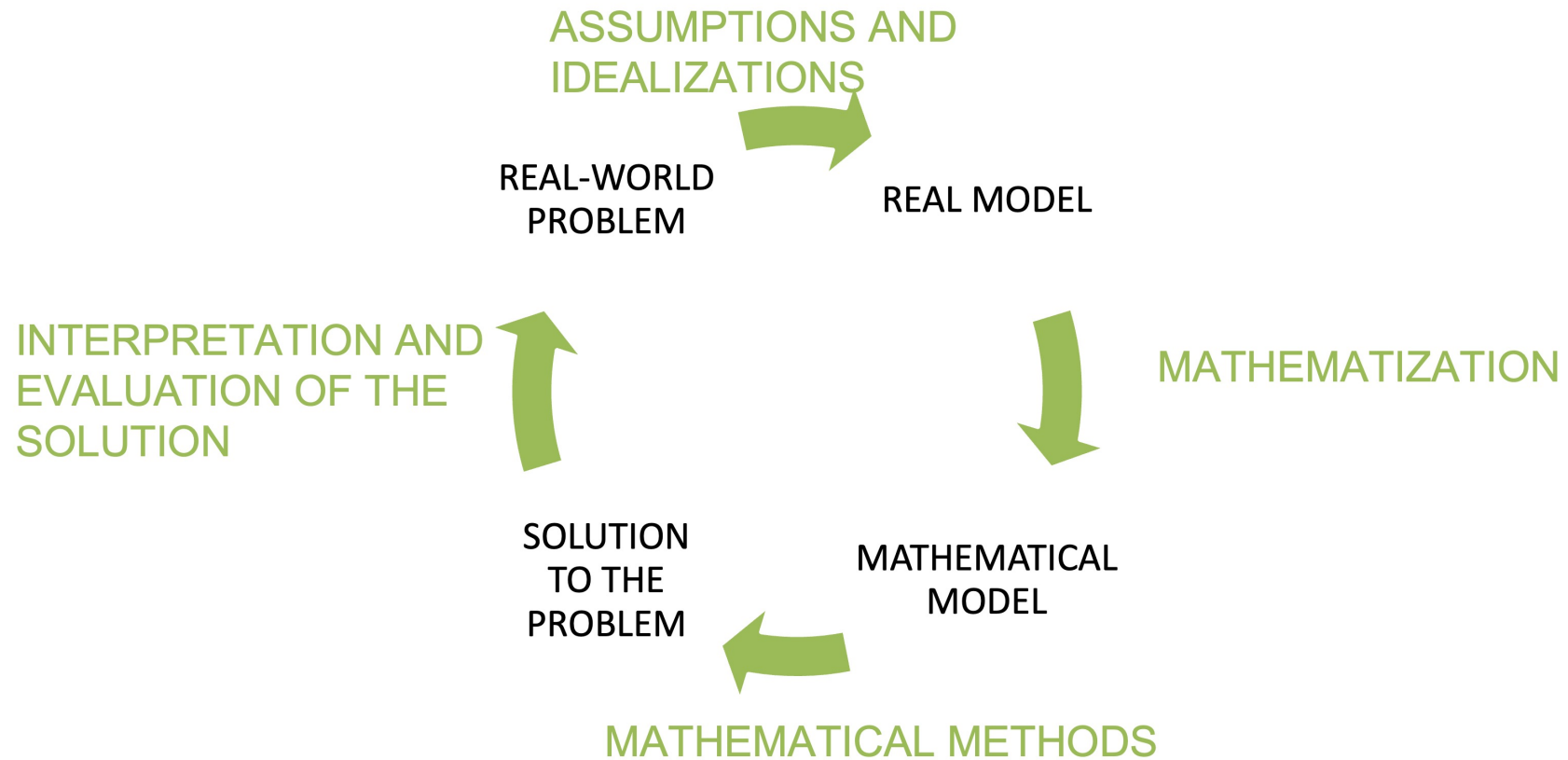
Parachute jump

Consider the problem of modelling *a parachute jump*.

- What could be the motivation of jumper? What is their goal?
- If we want to make an idealized model, which aspects of the jump can be neglected and which are important to keep?
- Which assumptions lead to the physical and mathematical description of the situation?
- Which dependencies could you describe in this situation? For example, can you describe the level of adrenaline in the blood of a jumper during the jump?



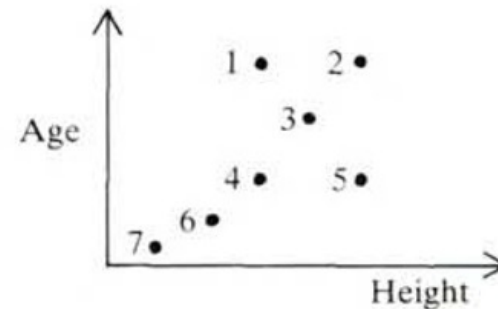
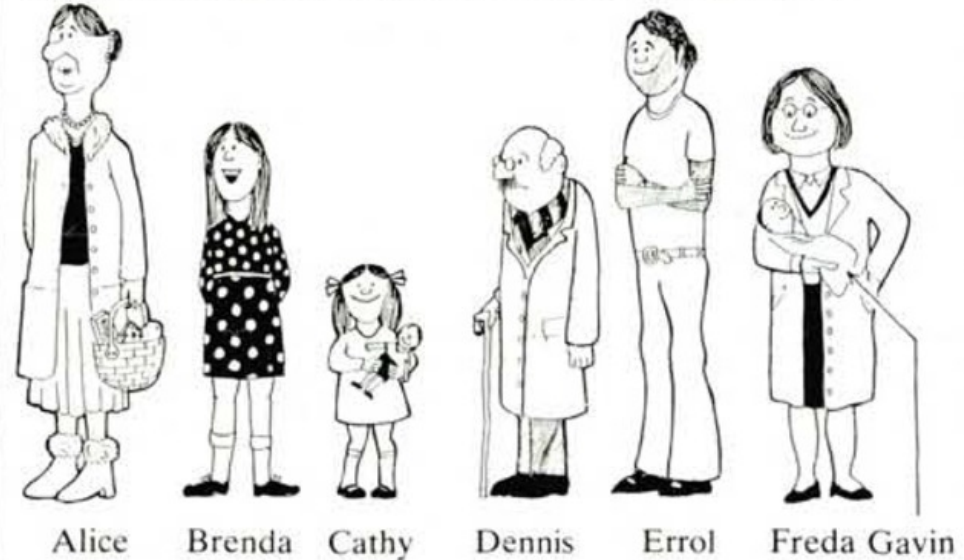
Mathematical modelling



Concept of a function

- Is it a function or a relation?
- Or both?
- Find examples of functions in your everyday life

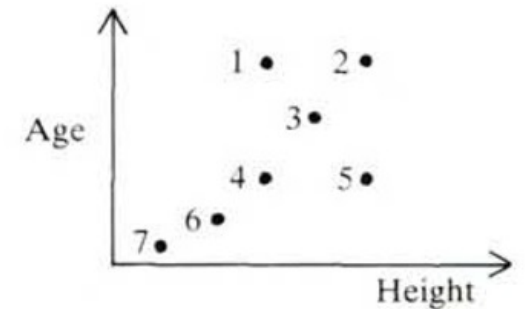
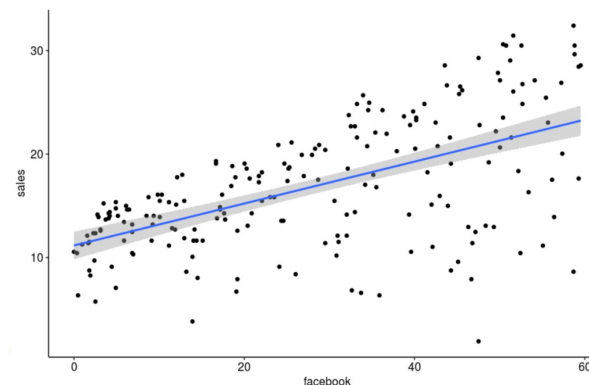
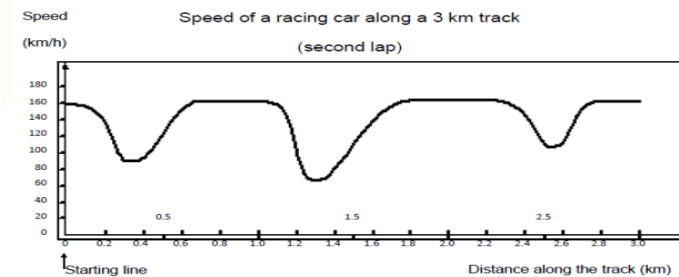
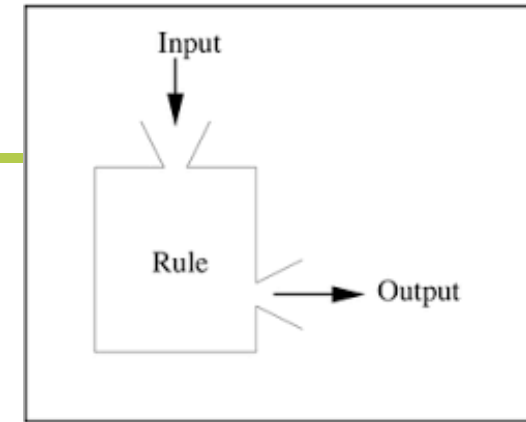
Who is represented by each point on the scattergraph, below?



Functional thinking

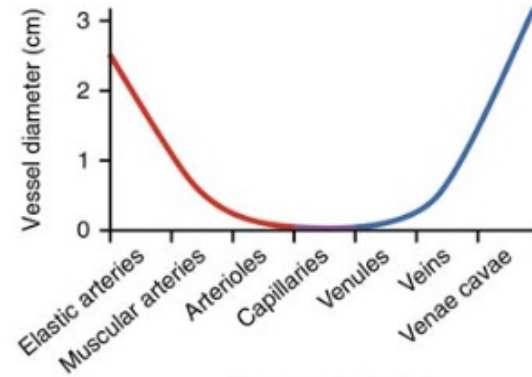
Four stages of developing functional thinking:

- Input-output action
- Covariational process
- Relational correspondence
- Part of a structure/scheme

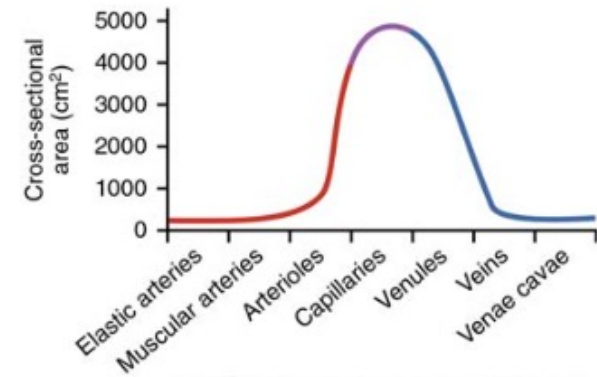


Interpreting graphs

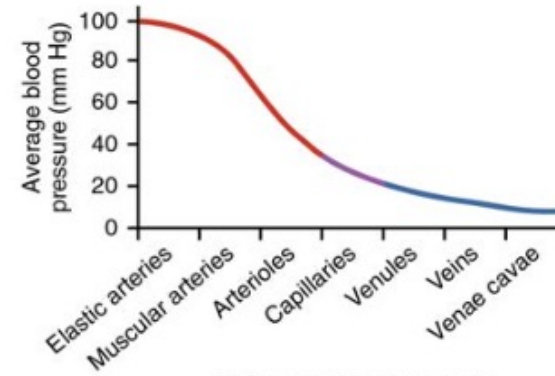
Describe the graphs in words.



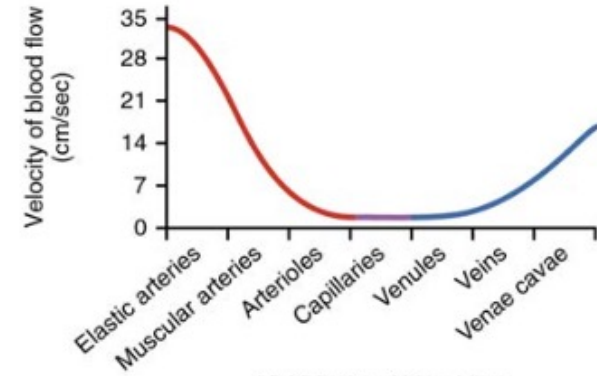
(a) Vessel diameter



(b) Total cross-sectional area of vessels



(c) Average blood pressure

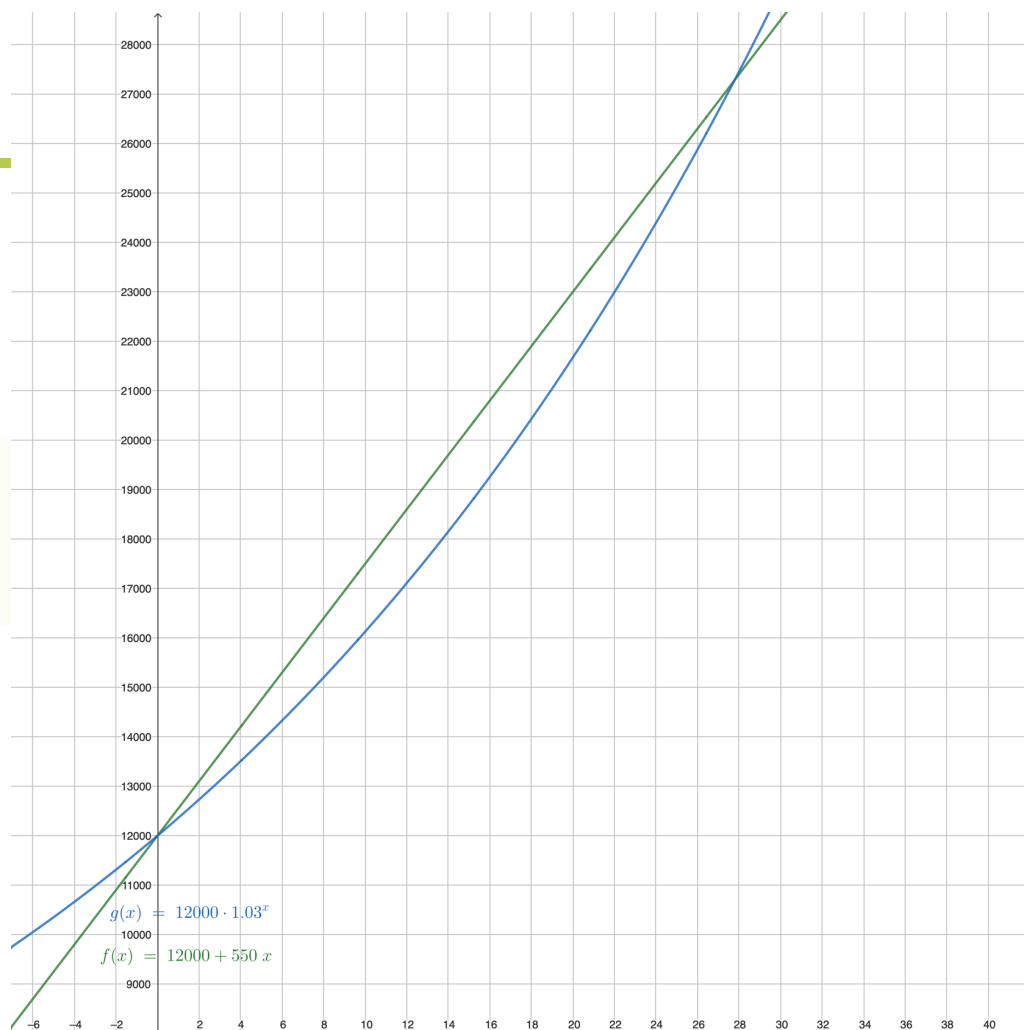


(d) Velocity of blood flow

Comparing growth

x	Andy (12000+550x)	Barbara (12000·1.03 ^x)
5	14 750	13 911
10	17 500	16 127
15	20 250	18 696
20	23 000	21 673
25	25 750	25 125
30	28 500	29 127

What might be surprising to students?
Was it surprising to you at first?
Is it possible to calculate the intersection points
of the two graphs?



Linear models

- It is easy for doctors to remember:

The mass of an infant from birth to the end of the first year of life triples.

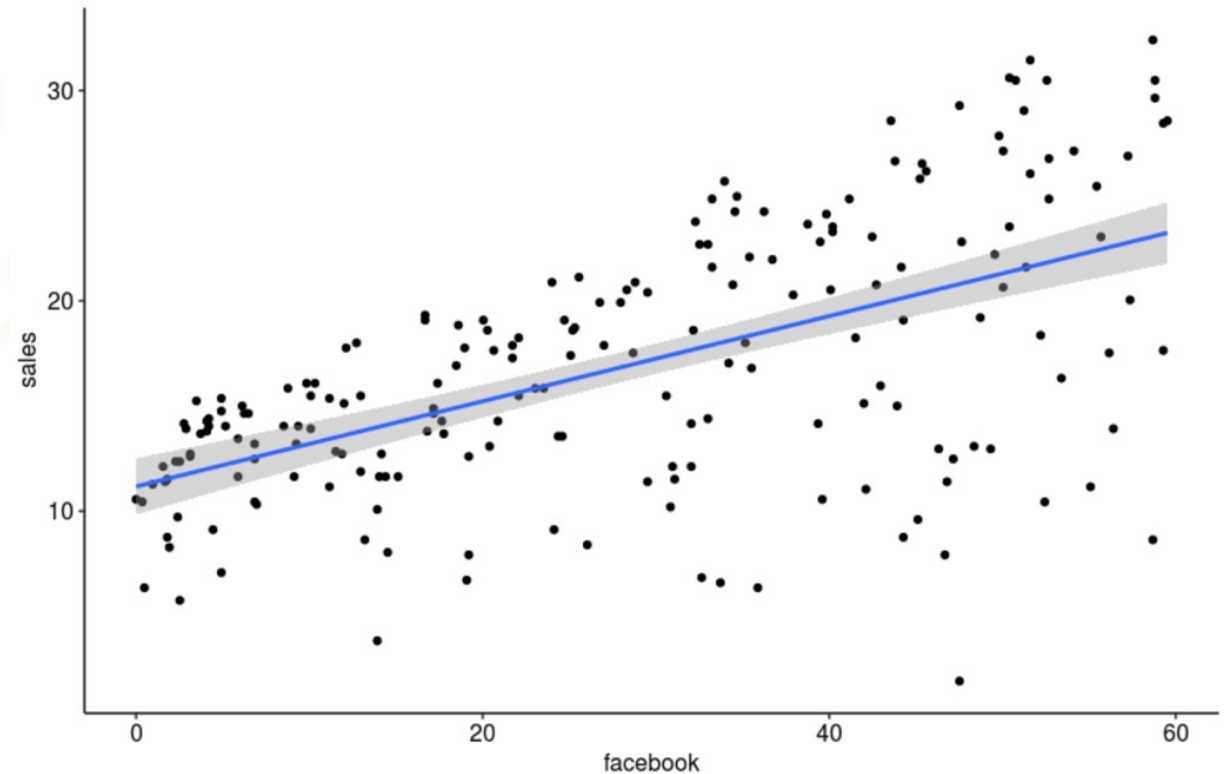
- Is it also valid afterwards?
- Can the length / height of the infant be assessed in this way?
(For a while we call it length, and then the child walks, so we call it height...)

The models are used for assessment, prediction... We need simple models!

Linear models - how do we make predictions?

Imagine you want to consider how your company's sales depend on the investment you make into advertising.

What is the effect of Facebook advertising on the company's sales, given the effects of YouTube and newspaper advertising?



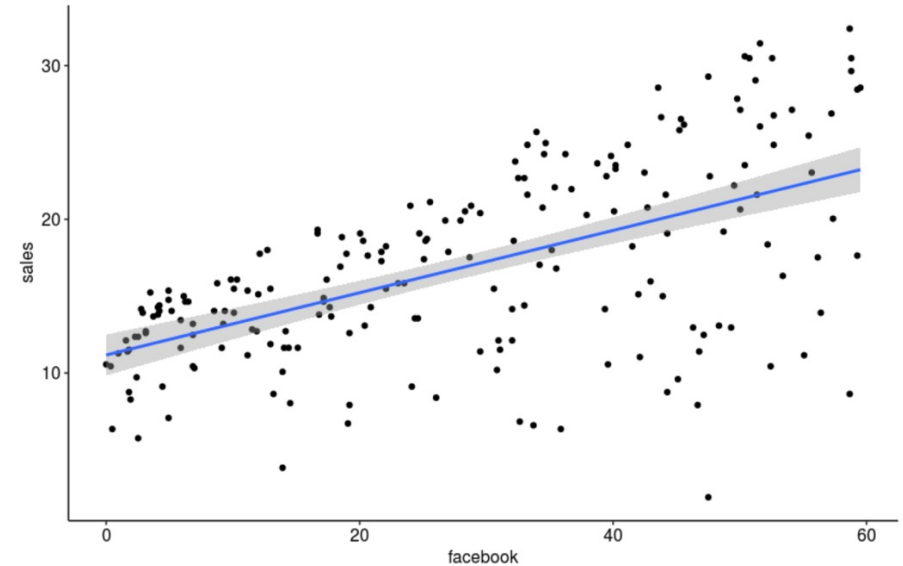
Linear models - how do we make predictions?

- How do you introduce linear models? Do you describe non-linear growth in words?
- How do you motivate the problem of making predictions using linear models?
- How do you arrive to the point that one needs to optimize the function

$$F(a, b) = \sum_{i=1}^n (y_i - ax_i - b)^2$$

and how do you perform optimization?

- Do you teach the least square method?



Drawing graphs from tables

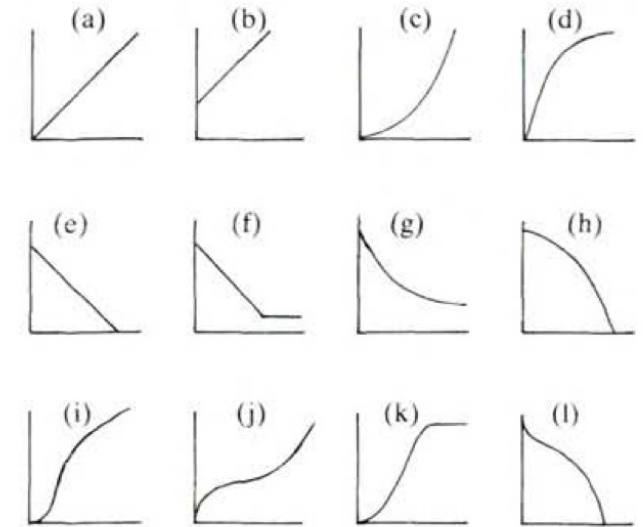
Time (hours)	1	2	3	4	5	6	7
Alcohol in blood (mg/100 ml)	90	75	60	45	30	15	0

Year	1880	1890	1900	1910	1920	1930	1940
Types of birds	0	1	5	17	30	30	30

Time (minutes)	0	5	10	15	20	25	30
Coffee temperature ($^{\circ}\text{C}$)	90	79	70	62	55	49	44

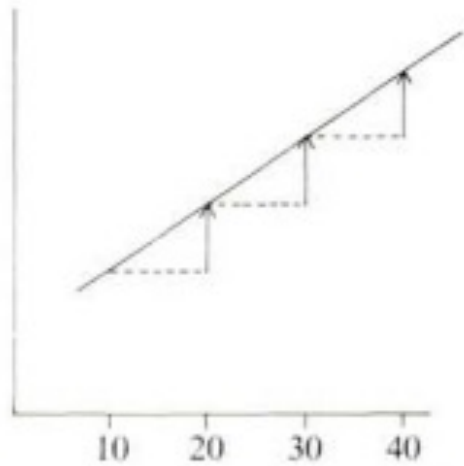
Turkey weight (kg)	3	4	5	6	7	8	9
Baking time (h)	2,5	3	3,5	4	4,5	5	5,5

Age (month)	2	3	4	5	6	7	8
Fetus length (cm)	4	9	16	24	30	34	38

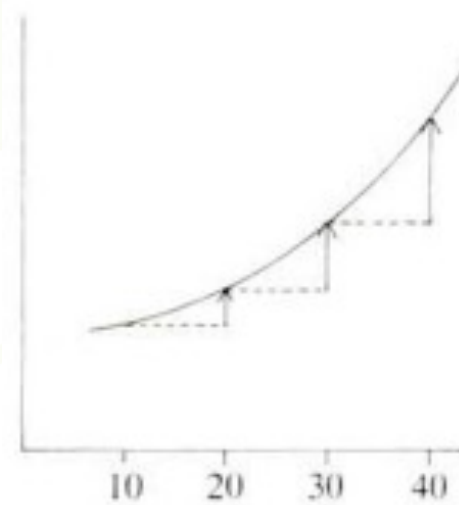


Connect the tables with the corresponding graphs!
Explain your reasoning.

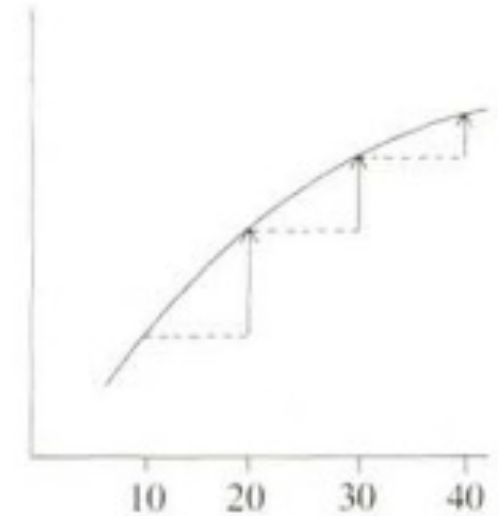
Linear and non-linear growth



Linear growth



Faster than linear growth



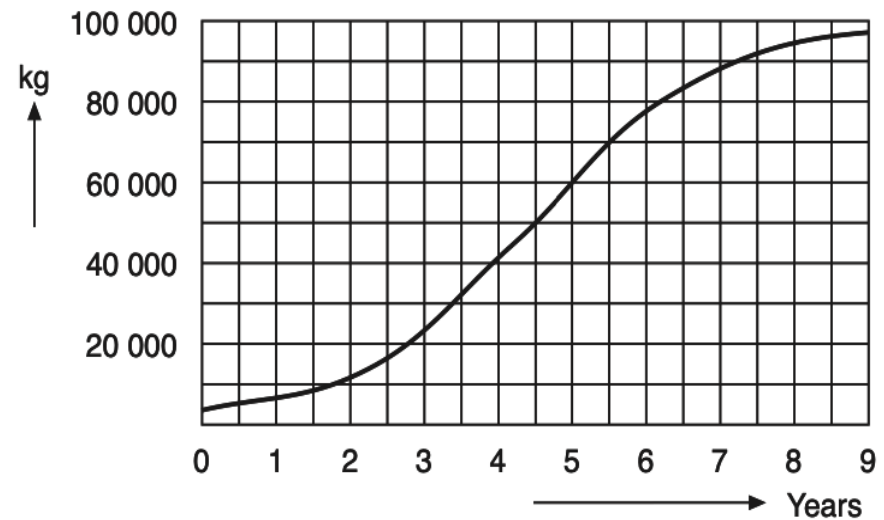
Slower than linear growth

Population growth

Imagine you are a fisherman that wants to grow fish in a waterway. You know that it takes some time for the population to grow, so you will wait a number of years and then start catching fish from the waterway. You will catch fish each year, hopefully for many years.

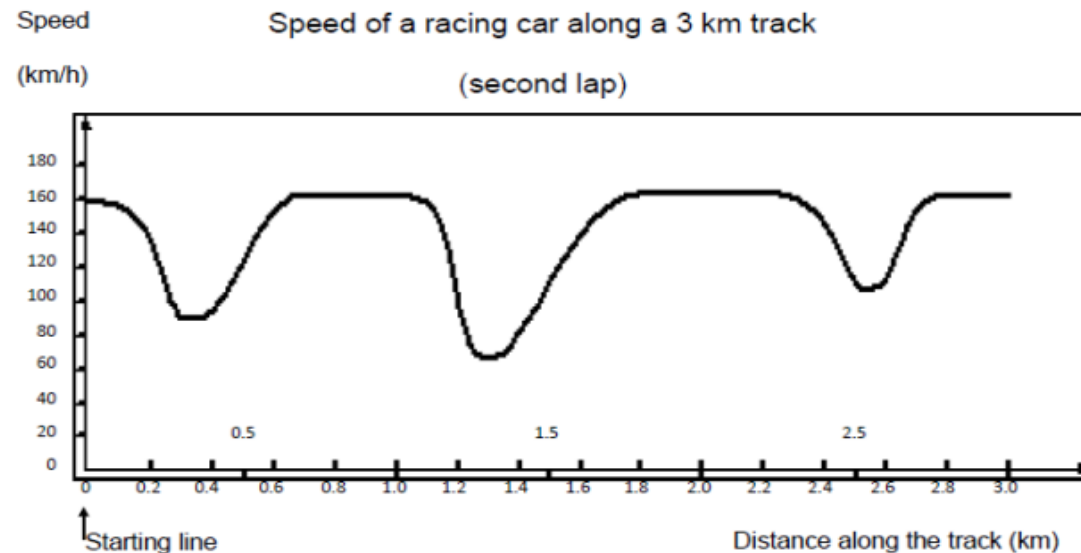
The graph shows a model of the growth in the combined weight of fish in the waterway.

How many years should the fisherman wait if he or she wishes to maximise the number of caught fish?



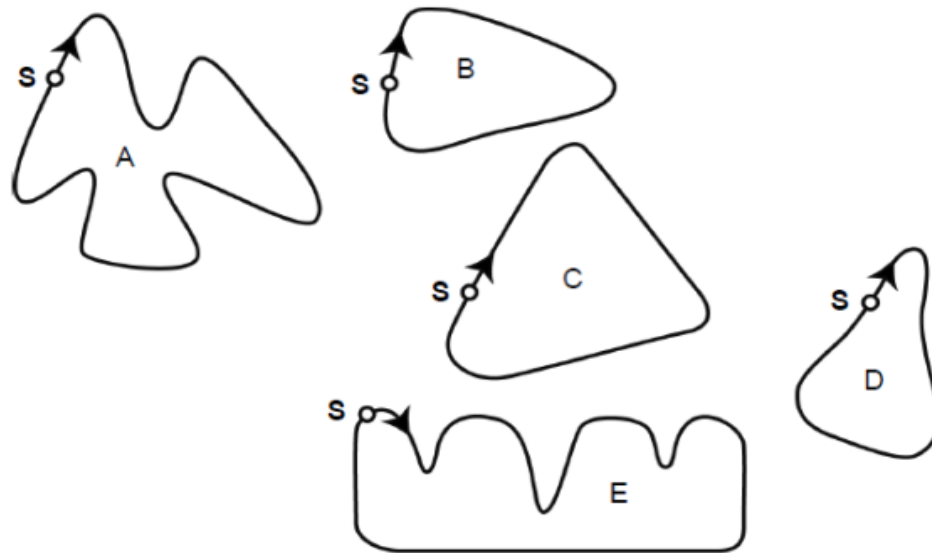
Racing car – even more graphs

This graph shows how the speed of a racing car varies along a flat 3 kilometer track during its second lap. What is the approximate distance from the starting line to the beginning of the longest straight section of the track?



Racing car

Here are pictures of five tracks. Along which one of these tracks was the car driven to produce the speed graph shown earlier?



S: Starting point

Racing car

For each track appearing on the left-hand side sketch a sensible speed-graph.

