STEMkey

Teaching standard STEM topics with a key competence approach

STEMkey Module 2

Functions

Worksheets

Co-funded by the Erasmus+ Programme of the European Union

This Worksheet document is based on the work within the project “Teaching standard STEM topics with a key competence approach (STEMkey)”. Coordination: Prof. Dr. Katja Maaß, International Centre for STEM Education (ICSE) at the University of Education Freiburg, Germany. Partners: Charles University, Constantine the Philosopher University, Haceteppe University, Institute of Education of the University of Lisbon, Norwegian University of Science and Technology, University of Innsbruck, University of Maribor, University of Nicosia, Faculty of Science of the University of Zagreb, Utrecht University, Vilnius University.

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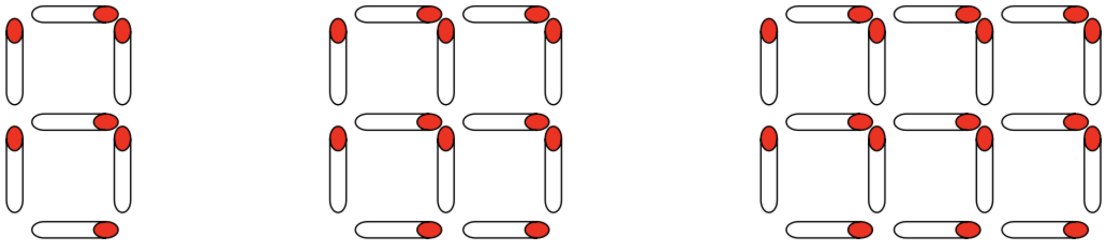
Worksheet 1.1 – Finding a pattern



Here are three shapes consisting of some number of matches.

If we continue to build further elements of this sequence how many matches would there be in the tenth shape? What about in general?

Discuss:

How to form different sequences of shapes and with which aims?

What is the role of inductive reasoning and at what age is the task appropriate?

What are the different **strategies** and **patterns** that one can use to solve this task?

Explain how this kind of exercises builds functional thinking and creativity.

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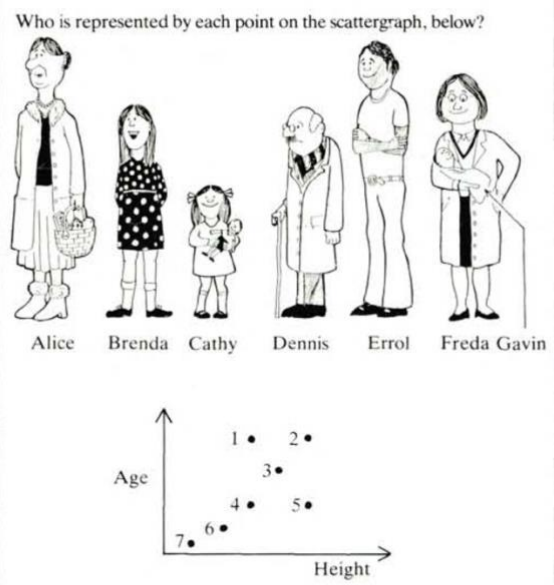
Worksheet 1.2 – Relations and functions



Source: The Language of Functions and Graphs, Shell Centre for Mathematical Education Publications, 1985.

Solve the above problem. Discuss in pairs how does this problem introduce the notion of a function.

What **strategies** would you expect from the students?

Are your students accustomed to thinking in **discrete or continuous** examples?

Can you make up more examples to discuss function properties, e.g. injectivity?

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Worksheet 1.3 – Parachute jump

Consider the problem of modelling *a parachute jump*.

What could be the motivation of jumper? What is their goal?

If we want to make an idealized model, which aspects of the jump can be neglected and which are important to keep?

Which assumptions lead to the physical and mathematical description of the situation?

What kind of graphical descriptions do you expect students could make?

Which dependencies could the students describe in this situation?

For example, can you describe the level of adrenaline in the blood of a jumper during the jump?

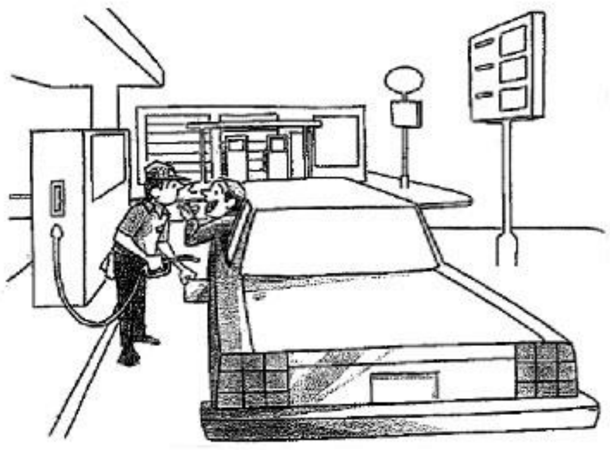
Worksheet 1.4 – Which gas station?

Consider buying gas at two different gas stations. One gas station has higher prices, but it is on our typical way from home to work. The second is not on that way and requires a detour but has cheaper gas price.

Which parameters would you consider?

Can you write a function that calculates the efficiency of buying gas with a detour?

Can you make the situation more concrete and draw a graphical representation?

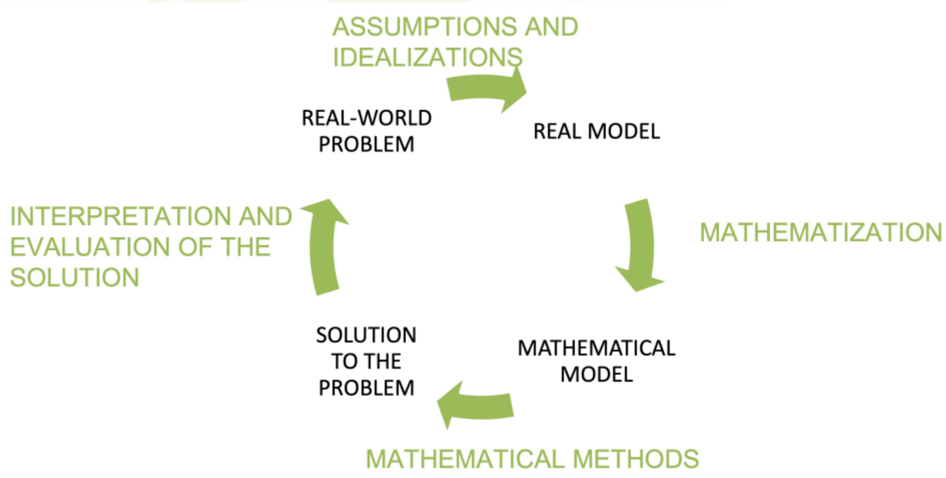


"Just enough to get me across the street to the cheaper station."

(Parade, 12 Nov 2005)

Which **phases of the modelling cycle** can be omitted (because they are already given) if we deal with the following task:

Frank usually buys gas at the station on his route between home and work at the price of 2.00 EUR per liter. At a station located 5 kilometers from his route gas is sold for 1.80 EUR per liter. Does it pay off to make a detour from his usual route to buy cheaper gas?



Worksheet 2.1 – Drawing graphs based on verbal descriptions

The students are asked to use the following axes to illustrate the sentence "*The more people we get to help, the sooner we'll finish picking the strawberries*." The graphs should be compared and discussed among peers.

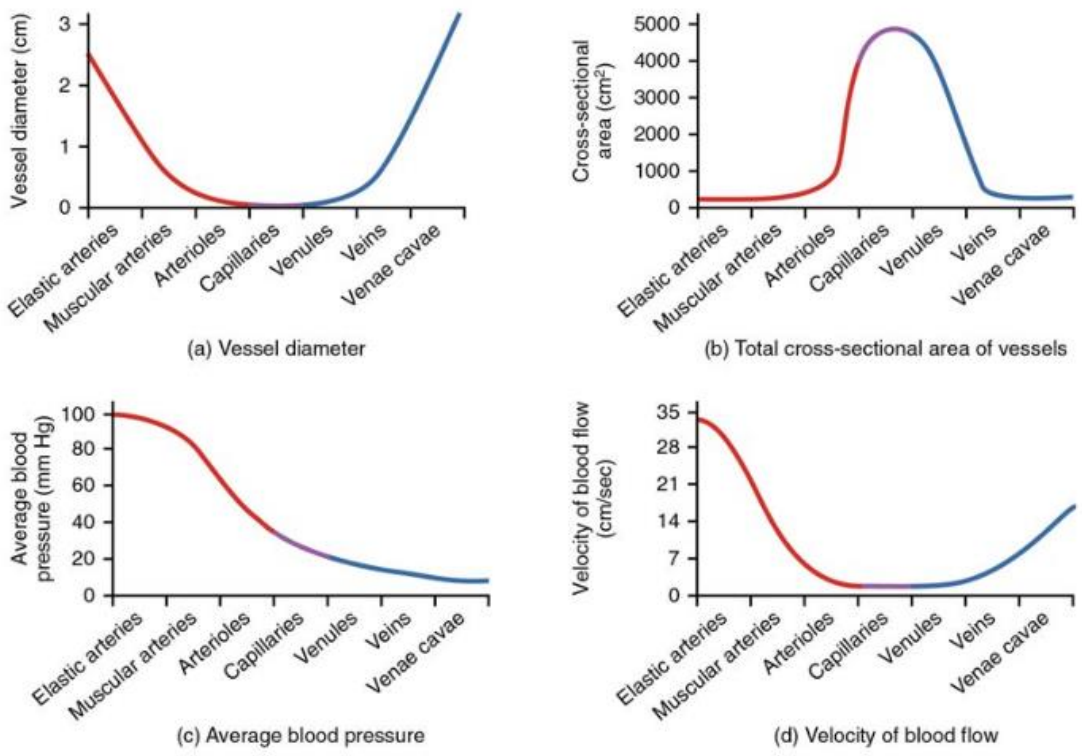
The task itself asks the students to compare their solution with peers and to be critical about it. It is expected that the students will anchor themself in the conception that a graph of a function should be a line, i.e. to insist on 'linear thinking'. By overcoming this obstacle, the students should realize that there are many useful functions which are not linear.

Do you ever pose a task to draw a graph describing a real-world situation?

Make up new sentences that will provide non-linear graphs – think about different contexts such as sports, cooking, sales etc. The situations described by graphs do not need to be numerical, you can ask to draw emotional states, tiredness or hunger over a time period.

Worksheet 2.2 – Interpreting graphs

Graphs of functions can be very informative, but one needs to know how to read them. The task is to read information from a graph in which the values on the x-axis are given by systematic vessels in a human body. Graphs should be **explained in words**.



Source of the picture: *Blood Flow, Blood Pressure and Resistance*, Anatomy and Physiology II, course on Lumen Learning. Link: https://courses.lumenlearning.com/suny-ap2/chapter/blood-flow-blood-pressure-and-resistance-no-content/

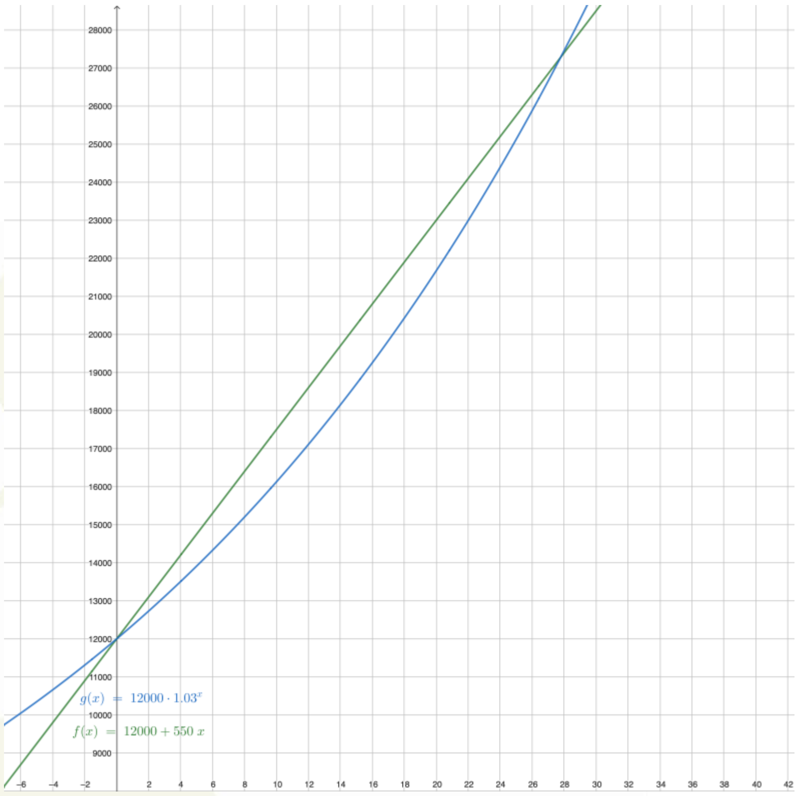
Worksheet 2.3 – Comparing growth

Discuss the following problem:

In 2018 Andy had 12000 EUR. Each year he added 550 EUR to the account. In 2018 Barbara had 12000 EUR. Each year she received interest rate of 3% on her investment (as she kept it in the account). Calculate the amount of money they each year until 2030, write a general formula for the amount they will have after x years and compare the growth.

x	Andy ($12000+550x$)	Barbara ($12000 \cdot 1.03^x$)
5	14 750	13 911
10	17 500	16 127
15	20 250	18 696
20	23 000	21 673
25	25 750	25 125
30	28 500	29 127

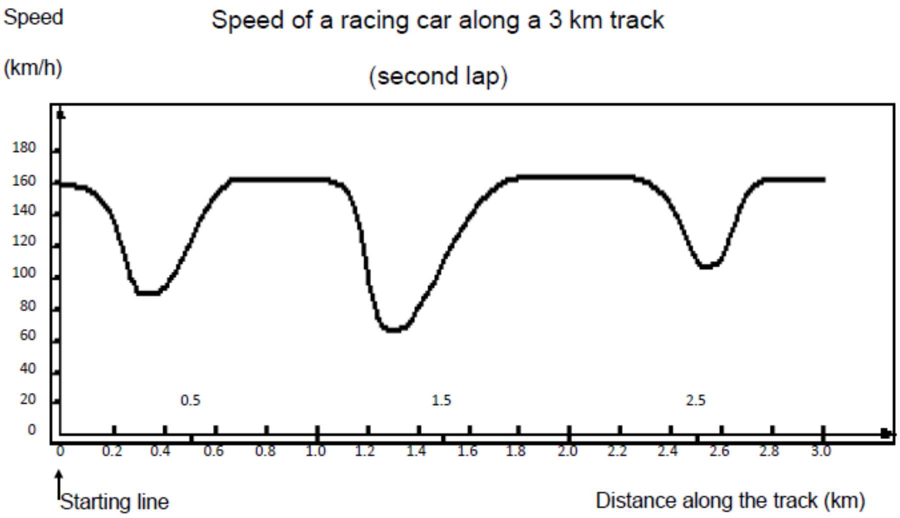
Compare the growth of two functions in the drawn interval.



What might be surprising to students? Was it surprising to you at first? Is it possible to calculate the intersection points of the two graphs?

Worksheet 2.4 – Racing car

This graph shows how the speed of a racing car varies along a flat 3 kilometer track during its second



lap.

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?

A. 0.5 km
B. 1.5 km
C. 1.8 km
D. 2.3 km
E. 2.6 km

Explain your thinking:

Where was the lowest speed recorded during the second lap?

A. At the starting line.
B. At about 0.8 km.
C. At about 1.3 km.
D. Halfway around the track.

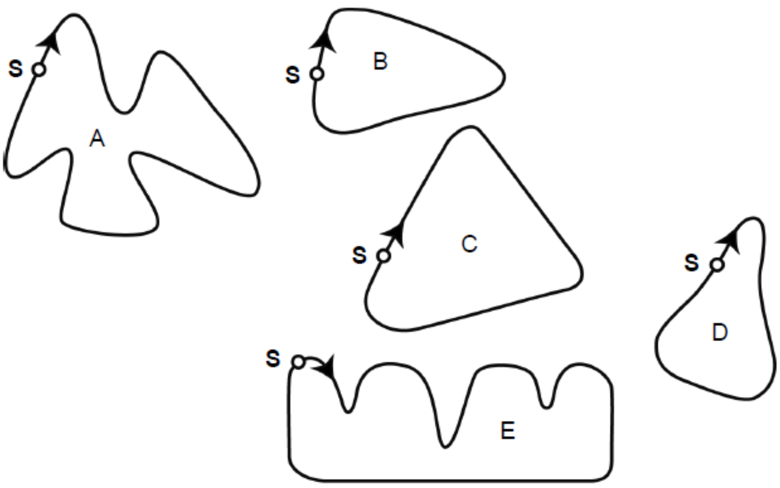
Explain your thinking:

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?

A. A The speed of the car remains constant.
B. B The speed of the car is increasing.
C. C The speed of the car is decreasing.
D. D The speed of the car cannot be determined from the graph.

Explain your thinking:

Here are pictures of five tracks. Along which one of these tracks was the car driven to produce the speed graph shown earlier?



S: Starting point

Explain your thinking:

For each track appearing on the left-hand side sketch a sensible speed-graph.

Worksheet 2.5 – Racing car – critical reflection

Evaluation goal: to explore ways to assess students critical thinking while solving a complex task in a real-world context. See IO1 for information on the rubric for Critical Thinking and its underpinning.

This activity is based on the activity called Racing car. The students have the task to study different racing tracks and associate them with the graph that describes the dependency of the speed along the track. How would you evaluate students' approach and reasoning to the task? Use the following rubric.

Neutral level

- *The naïve solution that the graph is curved at the places where the track is curved*
- *Misinterpreted variables on the axes of the graph*
- *No perspective of the driver of the car*
- *Sloppy graphs*
- *Drawing obviously wrong graphs*

Basic level

- *Applying a strategy that works only for some tracks that are similar to a shown example*
- *Explicitly linking the curvature of the graph with the curvature of the track*
- *Neat graphs, although not always correct*
- *Description of the reasoning in written text*
- *Evident change of a graph during the solving process*
- *Comparing the graph and concluding that it is or is not correct*

Proficient level

- *Evidence of at least two ways of explaining the shape of the graph,*
- *General patterns such as constant curvature of the track leads to constant speed*
- *Providing explanations considering practical and psychological aspects of a race*
- *Neat and correct graphs*
- *Confidence in the graphs and the reasoning behind them*

Expert level

- *Making up and discussing different types of tracks and graphs, not given by any source*
- *Providing explanations for efficiency of the method that directly connects the curvature of the track and the graph*
- *Expressing the value of functional thinking*
- *Discussing the difference of time-dependence and space-dependence of the speed*
- *Discussing possible misconceptions or pitfalls that beginners might encounter*

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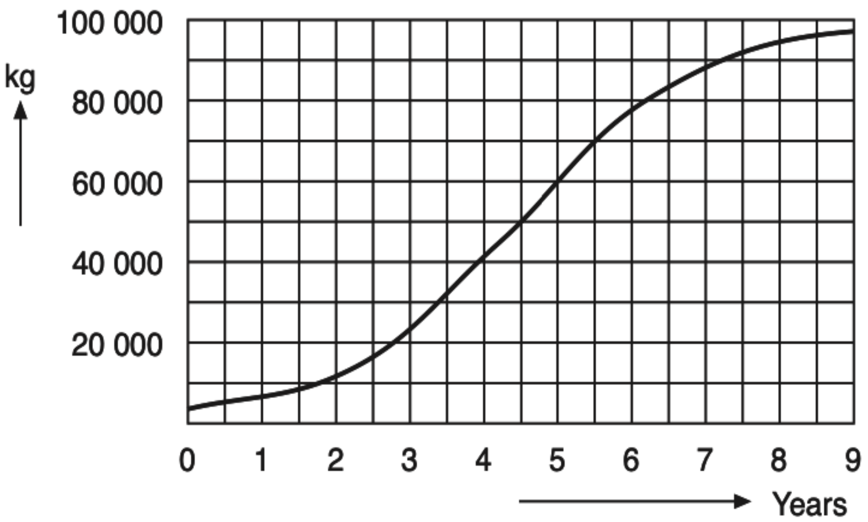
Worksheet 2.6 – Fish growth population

Imagine you are a fisherman that wants to grow fish in a waterway. You know that it takes some time for the population to grow, so you will wait a number of years and then start catching fish from the waterway. You will catch fish each year, hopefully for many years.

The graph shows a model of the growth in the combined weight of fish in the waterway. How many years should the fisherman wait if he or she wishes to maximize the number of caught fish?



What are the characteristics of this task?

Describe the most subtle moment of solving the task and how you would explain it as a teacher. How would you orchestrate a discussion that would allow students to form the explanation themselves?

Source of the picture: Measuring Student Knowledge and Skills - A New Framework for Assessment, OECD Programme for International Student Asseessment, 1999.

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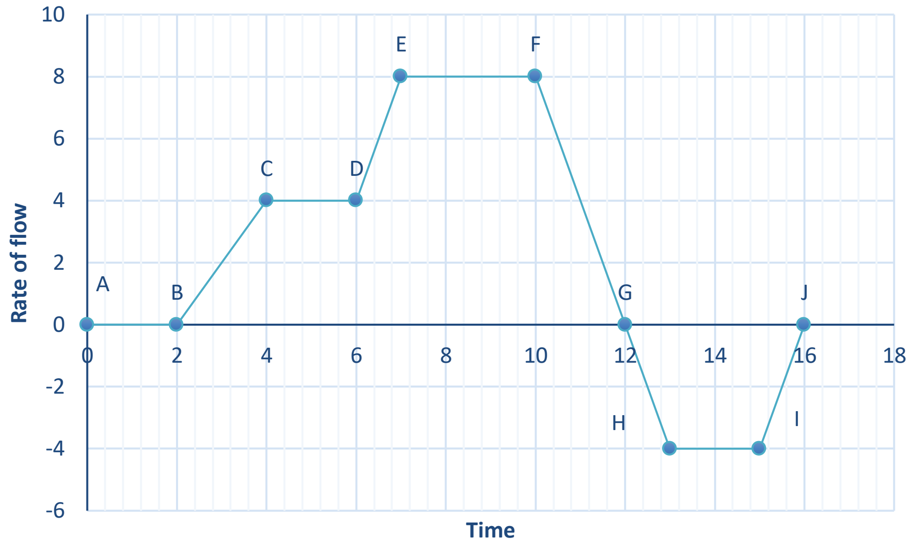
Worksheet 2.7 – The rate of flow

The figure shows the rate at which the water flows into or out of a container at different times.



When does the water volume increase and when does it decrease?

At what speed?

How do we connect the rate of flow and the total volume?

In which lesson units do you see that you could use this task? With which goals?

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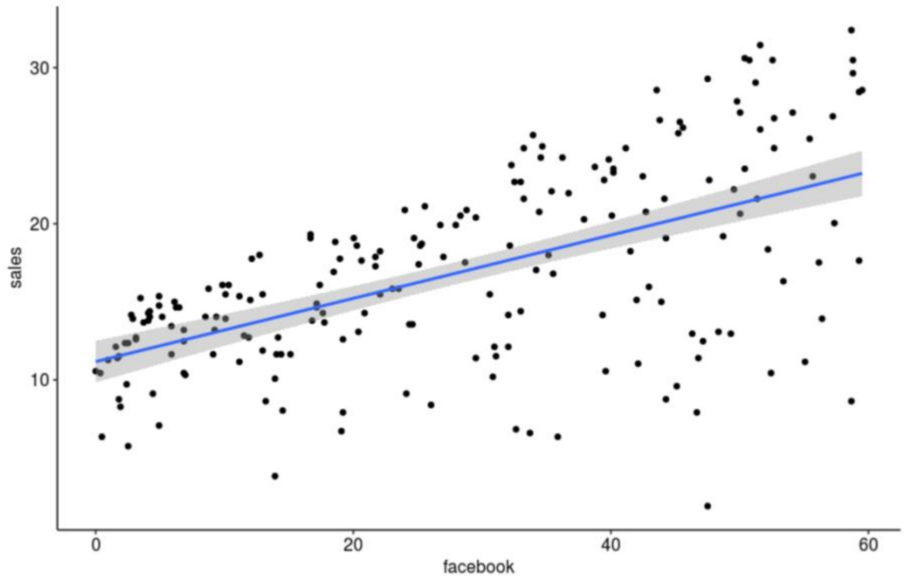
Worksheet 3.1 – Linear regression

Consider the following problem:

Imagine you want to consider how your company's sales depend on the investment you make into advertising. What is the effect of Facebook advertising on the company's sales, given the effects of YouTube and newspaper advertising?



Consider more data from the following link**:** https://towardsdatascience.com/predicting-the-impact-of-social-media-advertising-on-sales-with-linear-regression-b31e04f15982

How do you introduce linear models? Do you describe non-linear growth in words?

How do you motivate the problem of making predictions using linear models?

How do you arrive to the point that one needs to optimize the function

$$F(a,b) = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

and how do you perform optimization?

Do you teach the least square method? Do you use the "balance principle" or some other form of heuristic explanation to reason why the average should lie on the regression line?

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