Quarterly Problem

- *Math Edition* -

Pick's Theorem

Determining areas of irregular shapes like countries and regions can be complicated. However, a theorem discovered in 1899 by Austrian mathematician Georg Alexander Pick (1859–1942) can help to get simple and good approximations!

To use Pick's Theorem, one needs a square grid. The area of the smallest square in the grid shall be called the square unit. The set of points of intersections of the lines forming the grid is called a (square) lattice. Pick's theorem is applicable to any polygonal shape whose all vertices are lattice points – a lattice polygon.

Pick's Theorem: Given a lattice polygon, let I be the number of lattice points inside the polygon, and B the number of lattice points on its boundary. Then the area A of the polygon is $I + B/2 – 1$ square units.

Example **1.** Assuming the grid consists of unit squares (side length equal to 1), the area of the blue lattice polygon in the picture on the right is $47 + 9/2 – 1 = 50.5$ because there are 9 lattice points on the boundary of the polygon (marked red) and 47 lattice points inside.

Example **2.** The picture on the top of this page represents the map of Croatia, overlaid with a square grid. The square unit is 2500 km^2. The shape of Croatia is approximated by a lattice polygon (red). There are 20 lattice points on its boundary and 12 lattice points inside, so by Pick's formula its area is 21 square units, i.e., 52500 km^2. Note that even if the lattice polygon was not too good an approximation of Croatia's shape, we obtained a reasonable approximation of the true area (56594 km^2).

> **How does the area of the blue polygon below change if the length of the side of the squares that form the grid changes to 1 cm, 5 m or 10 km?**
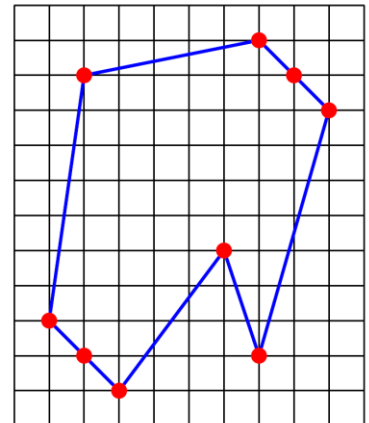
Choose a map of a country or region and calculate its area!

Draw a square grid over the chosen map, noting which length corresponds to the sides of the squares. Approximate the borders of the country by a lattice polygon and use Pick's theorem to calculate an approximation for the area represented by the map. Repeat with at least two different square grids with two different lattice polygon approximations each, and compare the results to the true area!

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